The Hasse Principle

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We are all familiar with the idea of finding the roots of polynomials. Our main concern for this talk will be finding a condition that is sufficient to show that a polynomial over $\mathbb{Q}$ has roots in $\mathbb{Q}$. It is not hard to see that a necessary condition is for the polynomial to have roots in $\mathbb{R}$. However, it is easy to find a polynomial over $\mathbb{Q}$ with roots in $\mathbb{R}$ but not in $\mathbb{Q}$. We will start by introducing the field of p-adic numbers, $\mathbb{Q}_p$. Our focus will be in polynomials over $\mathbb{Q}$ with roots in all $\mathbb{Q}_p$ and $\mathbb{R}$. Similarly to before, this will be a necessary condition, but will it be sufficient? This is known as the Hasse Principle. In general, this principle is not true, but there are stronger conditions which can force it to be true. We will consider a few of these conditions.

Prerequisites

First year undergraduate analysis (Math 316/317)

References

[1] Gouvêa, Fernando Q. *p*-adic numbers. An introduction. Second edition. Universitext. Springer-Verlag, Berlin, 1997.