Mathematics in Weak Axiomatic Systems

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Abstract: Relative to the vast history of mathematics, the development of axiomatized systems in which to prove theorems is a modern concept. When developing these systems, it is important to keep axioms consistent with each other, non-controversial, and not overly assumptious. There is then the question of how many axioms are actually needed for a system to prove the theorems we want it to. Peano Arithmetic (PA) is an attempt at a simplistic system in which to prove theorems, using the natural numbers as a model. We will introduce the Axioms of PA and as an example, we'll prove the commutativity of addition. There are even weaker systems than PA, such as Elementary Function Arithmetic (EFA), which weakens the induction scheme of PA. We will go on to illustrate the differences between PA and EFA.

Prerequisites: MATH 307 (Intro to Proof) or equivalent.

References

Goldstern, Martin and Judah, Haim. *The Incompleteness Phenomenon: a New Course in Mathematical Logic,* A K Peters, 1995.

Hájek, Petr and Pudlák, Pavel. *Metamathematics of First-Order Arithmetic,* Association for Symbolic Logic, 2016.