DRP Abstract

Derek J. DeRouen

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Title: Quadratic Reciprocity

Mentee and Mentor names:
Derek J. DeRouen, Mentee Sean Haight, Mentor

A brief summary of your talk topic:
Number theorists are often looking for integral solutions to equations. A first step is to investigate whether these equations have solutions modulo q for an odd prime q. The question this talk will focus on is, for odd primes p and q, does $x^2 \equiv p \pmod{q}$ have an integer solution. In order to accomplish this we must implore the use of a tool named the Legendre Symbol written $(\frac{a}{b})$ such that:

$$\left(\frac{a}{b}\right) = \begin{cases} 1 & \text{if a is a quadratic residue modulo p} \\ -1 & \text{if a is a quadratic non-residue modulo p} \end{cases}$$

This leads us to be able to state the Law of Quadratic Reciprocity as Follows:

Let p and q be distinct odd prime numbers. Then

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{(\frac{p-1}{2})(\frac{q-1}{2})}$$
$$= \begin{cases} 1 & \text{if } p \equiv 1 \pmod{4} \text{ or } q \equiv 1 \pmod{4} \\ -1 & \text{if } p \equiv q \equiv 3 \pmod{4} \end{cases}$$

We will discuss the prelims, proof, as well as the significance and applications.

Prerequisites required:

Elementary number theory is useful but not necessary.

Reference list:
Strayer, James K. Elementary Number Theory. Waveland Press, 2002.