Lecture 15
Networks: Small Worlds

Geog 490/590
Spatial Modeling
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Assembly of complex plant-fungus networks

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Species in ecological communities build complex webs of interaction. Although revealing the architecture of these networks is fundamental to understanding ecological and evolutionary dynamics in nature, it has been difficult to characterize the structure of most species-rich ecological systems. By overcoming this limitation through next-generation sequencing technology, we herein uncover the network architecture of below-ground plant-fungus symbioses, which are ubiquitous to terrestrial ecosystems. The examined symbiotic network of a temperate forest in Japan includes 33 plant species and 387 functionally and phylogenetically diverse fungal taxa, and the overall network architecture differs fundamentally from that of other ecological networks. In contrast to results for other ecological networks

Figure 1 | Architecture of the below-ground plant-fungus network in a temperate forest in Japan. In the bipartite network, plant species (red) interact with ectomycorrhizal (yellow) and arbuscular mycorrhizal (pink) fungal OTUs as well as OTUs with unknown ecological functions (blue). The size of nodes represents the relative abundance of plant species or fungal OTUs in the data set."
Individuals (nodes)
Relationships (links)
Network Clusters
Connected Network
Social Networks
Social Networks
Social Networks
Social Networks
Social Networks
(People) make their own history, but...they do not make it under circumstances chosen by themselves.
Random Network

fraction of all nodes in giant component

average number of links per node

Giant Component & p.46
Random Network

- fraction of all nodes in giant component
- average number of links per node

phase transition
Limits to the Random Network Approach?
Limits to the Random Network Approach

Are your connections random?
The Delicate Balance of Social Networks

**Structure**: the surrounding constraints in which we operate

**Agency**: individual preferences
It's a small world
you
you
you
Small World Networks

1. Consist of many, small overlapping groups
2. Dynamic
3. Not all relationships (links) are have an equal probability of occurring
4. Actors connect based on individual preference
Clusters vs Connected
Connection Preference

\[ a \text{ (alpha)} = \text{ preference for connection} \]
Connection Preference

\[ a \ (\alpha) = \text{preference for connection} \]

\[ \text{low } a = \text{preference to connect to friends of your friends} \]
Connection Preference

\[ a \text{ (alpha)} = \text{preference for connection} \]

low \( a = \) preference to connect to friends of your friends

high \( a = \) preference to connect to someone other than a friend of your friends
likelihood that A meets B

number of mutual friends between A and B

a = 0

a = infinity
likelihood that A meets B

number of mutual friends between A and B

a = 0

a = infinity
The likelihood that A meets B is shown as a function of the number of mutual friends between A and B. The diagram includes two labels: "a = 0" and "a = infinity," indicating different conditions or parameters that affect the likelihood. The vertical axis represents the likelihood of meeting, while the horizontal axis represents the number of mutual friends.
The likelihood that A meets B is a function of the number of mutual friends between A and B. As the number of mutual friends increases, the likelihood also increases. The parameter $a$ affects the shape of the likelihood function:

- $a = 0$ represents a specific case where the likelihood is determined by the number of mutual friends.
- $a = 1$ indicates a different form of the likelihood function.
- $a = \infty$ implies an even further modified version of the likelihood function.

The diagram illustrates these relationships, with different curves representing varying values of $a$. The $y$-axis represents the likelihood that A meets B, while the $x$-axis indicates the number of mutual friends between A and B.
Path Length

average number of steps along the shortest paths for all possible pairs of network nodes
Clustering Coefficient

a measure of the degree to which a single node cluster together (are your friends friends?)
Clustering Coefficient

c = 1

you
Clustering Coefficient

$\text{you}$

$c = \frac{2}{3}$
Clustering Coefficient

\[ c = \frac{1}{3} \]
Clustering Coefficient

\[ c = 0 \]

you
fragmented network
The Take Away

1. Networks can either be clustered or connected; they cannot be both.

2. The critical value of alpha represents a phase transition from a clustered to a connected (small world) network.

3. Once a network passes through the phase transition, it becomes amenable (or susceptible) to the spread of some entity throughout the network.
Team Assembly Model