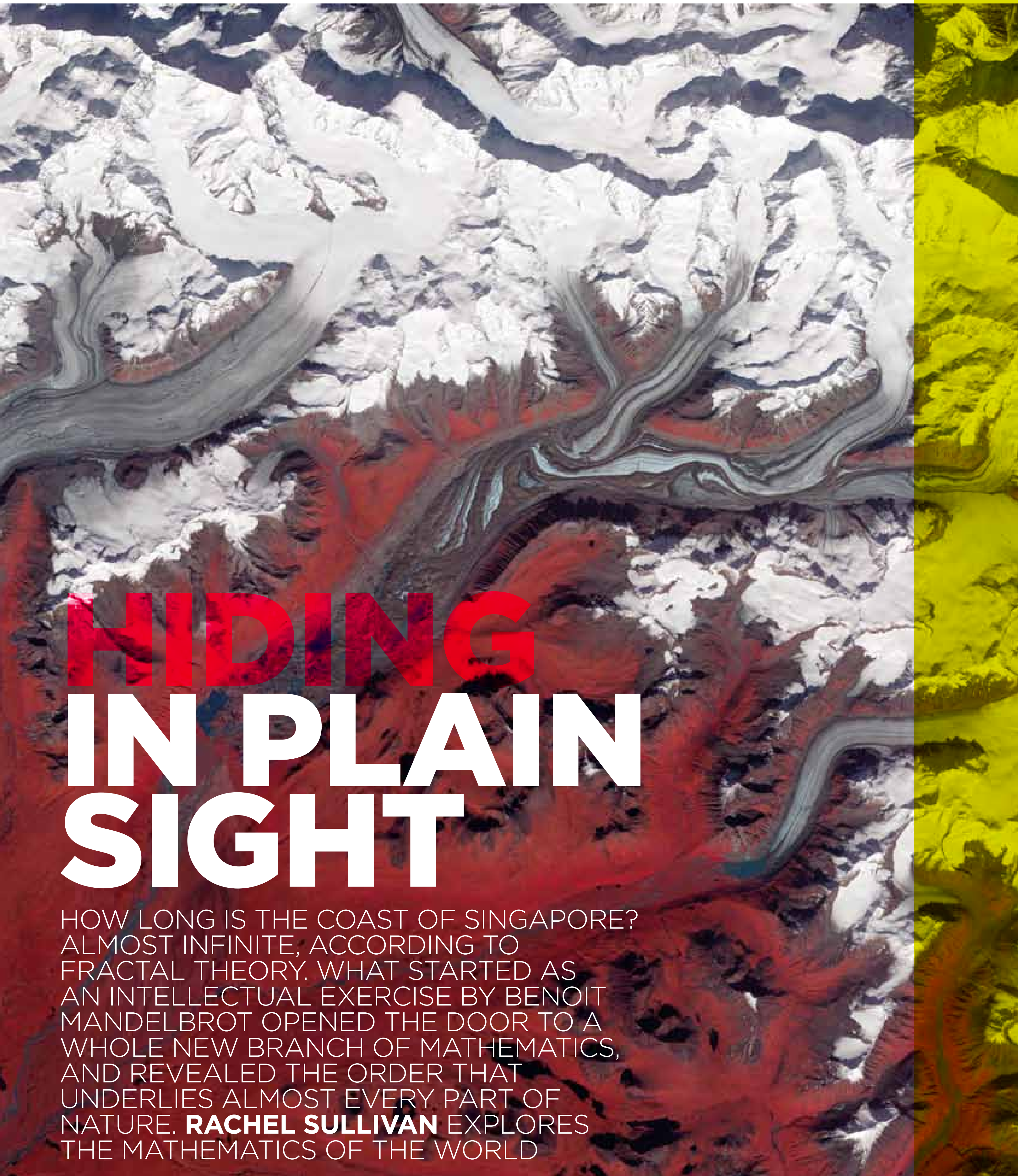
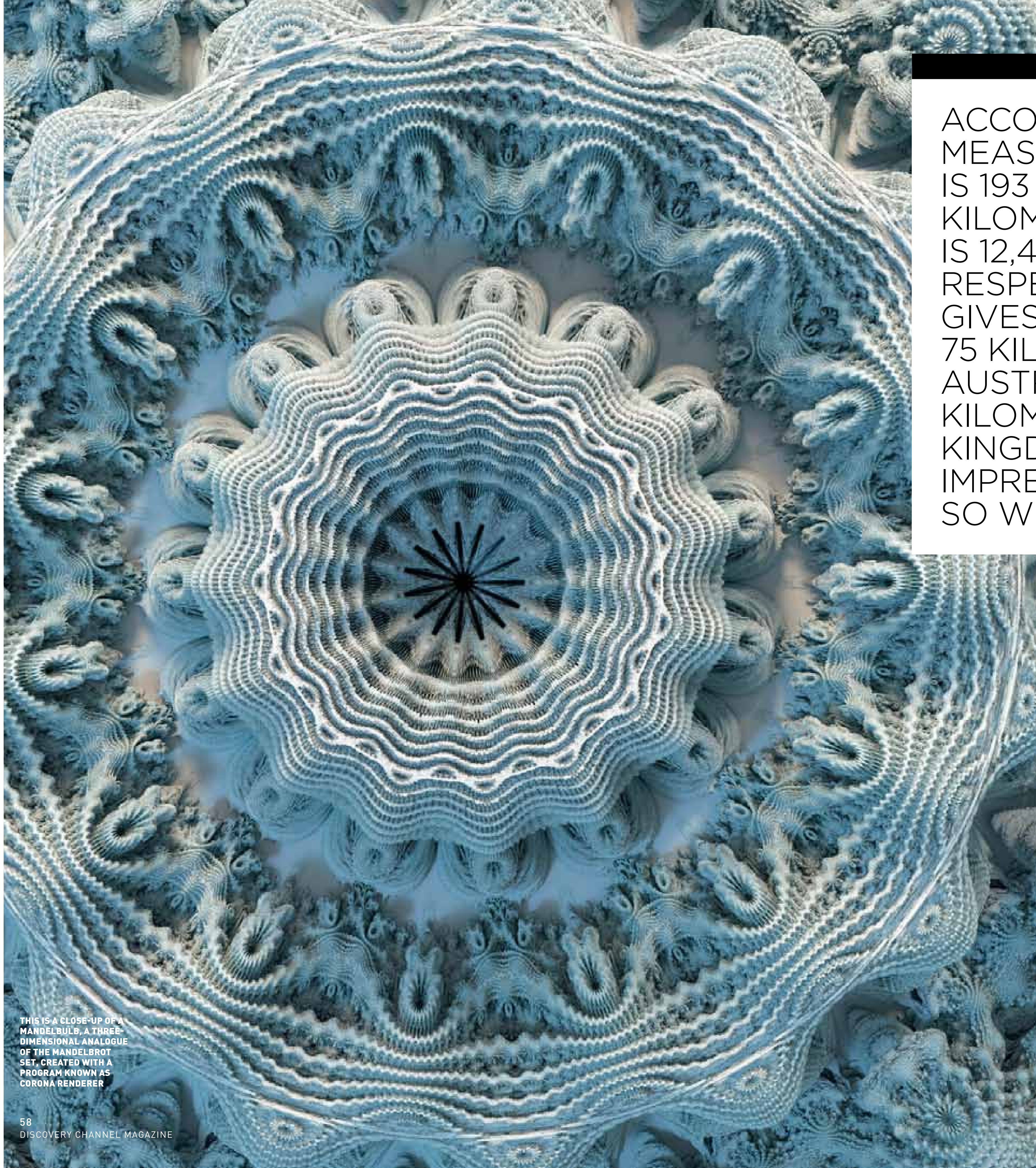


HIDING IN PLAIN SIGHT

HOW LONG IS THE COAST OF SINGAPORE? ALMOST INFINITE, ACCORDING TO FRACTAL THEORY. WHAT STARTED AS AN INTELLECTUAL EXERCISE BY BENOIT MANDELBROT OPENED THE DOOR TO A WHOLE NEW BRANCH OF MATHEMATICS, AND REVEALED THE ORDER THAT UNDERLIES ALMOST EVERY PART OF NATURE. **RACHEL SULLIVAN** EXPLORES THE MATHEMATICS OF THE WORLD

PHOTOS: NASA (OPPOSITE), CORBIS

WHETHER IT'S GLACIER MOVEMENTS (*OPPOSITE*) OR TREES, FRACTAL GEOMETRY IS FOUND ALMOST EVERYWHERE IN NATURE — KEEP YOUR EYES PEELED AND YOU'LL SEE IT IN SOME TRULY STRANGE PLACES



THIS IS A CLOSE-UP OF A MANDELBULB, A THREE-DIMENSIONAL ANALOGUE OF THE MANDELBROT SET, CREATED WITH A PROGRAM KNOWN AS CORONA RENDERER

PHOTO: ONDREJ KARLI

ACCORDING TO ONE RELIABLE SET OF MEASUREMENTS, SINGAPORE'S COASTLINE IS 193 KILOMETRES, AUSTRALIA'S IS 25,760 KILOMETRES, AND THE UNITED KINGDOM'S IS 12,429 KILOMETRES. ANOTHER EQUALLY RESPECTABLE SET OF MEASUREMENTS GIVES SINGAPORE A VALUABLE ADDITIONAL 75 KILOMETRES (FOR A TOTAL OF 268); AUSTRALIA GETS WHOPPING EXTRA 40,770 KILOMETRES TO ITS NAME; AND THE UNITED KINGDOM A MORE MODEST BUT STILL IMPRESSIVE 7,288 KILOMETRES EXTRA. SO WHY THE DISPARITY?

It all depends on the unit of measurement. If you measure in lots of 50 kilometres then you get the first set of measurements. But because a land mass has features at scales that range from hundreds of metres to the nanoscale, the smaller the unit of measurement, the longer the coastline will appear to be. So even though logically a coastline has a finite length, from a mathematical perspective it is infinite.

This paradox left cartographers scratching their heads, until Polish-born mathematician Benoit Mandelbrot brought a new approach to the table. In attempting to accurately measure the coastline of Great Britain, Mandelbrot found something that had apparently eluded conscious human minds for millennia — the numerical order that is in fact underlying nature's apparent chaos.

Mandelbrot couldn't measure the length of every nook and cranny either, but he did recognise that coastlines, like clouds, mountains, trees and many other natural objects, are made of irregular shapes that endlessly repeat themselves at different scales. He found a way to rate their roughness, and opened the door to a whole new branch of mathematics.

Unlike geometrically smooth surfaces such as one-dimensional lines, two-dimensional triangles or three-dimensional spheres, Mandelbrot found that so-called fractal shapes, irregular patterns that are reproduced infinitely at all scales to produce natural phenomena, fall somewhere between two and three dimensions. The rougher the surface, he discovered, the higher its fractal dimension.

Speaking at a TED talk shortly before his death in 2010, the Yale mathematician, a citizen of France and the US, used the example of a romanesco cauliflower to illustrate the concept of fractals, and of self-similarity. Self-similarity is one of fractals' most important underlying principles, and arises from endless repetition of a simple pattern.

"A cauliflower is both very complicated and very simple at the same time," he said. "If you cut one of the florets with a sharp knife and look at it, you see the whole cauliflower, but smaller. And then you cut again, and again, and again... and each time you see smaller cauliflowers. Humans have always recognised that some shapes have this peculiar property, where each part is like the whole, only smaller."

In geometric terms a coastline is a fractal, which remains similar-looking at all scales. Like the distance around a lake, he noted, "The concept of the length of a coastline should be straightforward. It is published in many places — but is in fact a complete fallacy because each time you get closer, the longer the distance becomes."

PRACTICAL DESIGN

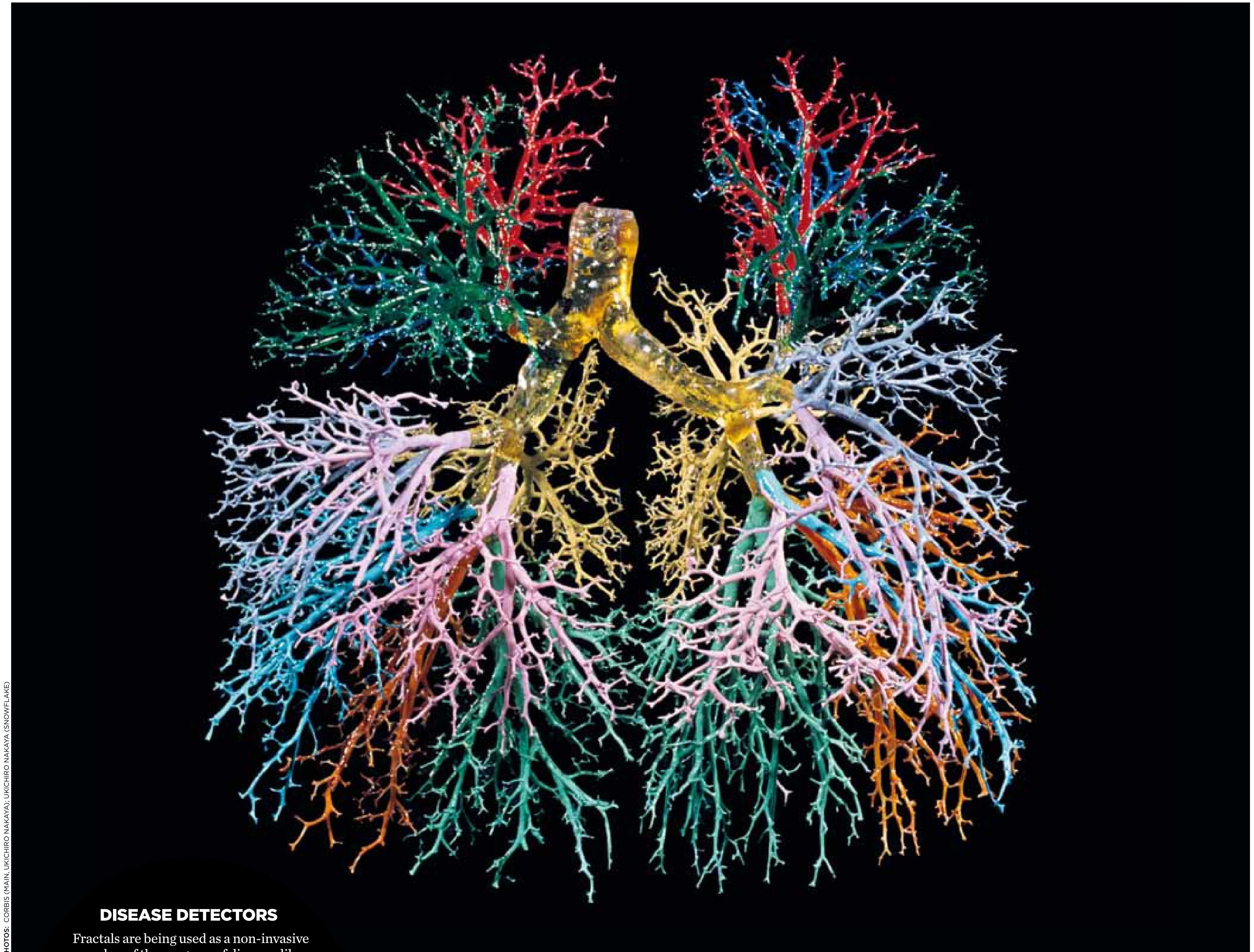
Fractal shapes are found everywhere in nature and at all scales, from the rings of Saturn and the spiral shapes of galaxies, to the beat of the human heart; from the branching of our blood vessels, nervous system and DNA, to the structure of mountain ranges, river deltas and seashells. Electrical impulses travel across wires in a fractal pattern, whether they are high-tensile wires or nanowires. And researchers in the US state of Arizona have found that the ratio of large trees to small ones in a forest exactly matches the ratio of large branches to small ones in the trees that make up the forest — and so on, even down to the cellular level.

FRACTAL SHAPES ARE FOUND EVERYWHERE IN NATURE AND AT ALL SCALES, FROM THE RINGS OF SATURN AND THE SPIRAL SHAPES OF GALAXIES, TO THE BEAT OF THE HUMAN HEART

"There are two types of fractals," explains Dr Richard Taylor, professor of physics, psychology, and art, and director of the Materials Science Institute at the University of Oregon in the United States, who is currently developing fractal energy circuits in retinal implants and solar cells.

"One is exact, where a pattern is repeated precisely at different scales such as the Mandelbrot set or Koch snowflake (*see page 65*)," he says. "The other is a natural fractal, which is more subtle. In natural fractals the pattern doesn't repeat precisely but the general quality, known as statistical self-similarity, is repeated at smaller and smaller scales."

Taylor notes, "Nature in the raw can be mind-bogglingly complex, but when you look beyond and think about how the complexity was created, it is remarkably easy."



PHOTOS: CORBIS (MAIN, UKICHIRO NAKAYA); UKICHIRO NAKAYA (SNOWFLAKE)

ABOVE: A RESIN CAST OF THE HUMAN LUNGS AND BRONCHIAL TREE. THE AIRWAYS OF EACH LOBE OF THE LUNG HAVE BEEN CAST IN A DIFFERENT COLOUR. THIS LARGE NETWORK OF AIRWAYS GIVES THE LUNGS A HUGE SURFACE AREA FOR OXYGEN TO ENTER THE BLOOD AND CARBON DIOXIDE TO EXIT

DISEASE DETECTORS

Fractals are being used as a non-invasive marker of the progress of diseases like Alzheimer's. Scientists recognise that the central nervous system and the activities it controls have an intrinsic fractal structure, with similar patterns, whether monitored in minutes or hours. These activity patterns change with age and disease, and Harvard scientists have recently discovered that as Alzheimer's disease progresses in a patient, the disruption to their fractal activity patterns increase, giving doctors additional insight into the insidious disease.

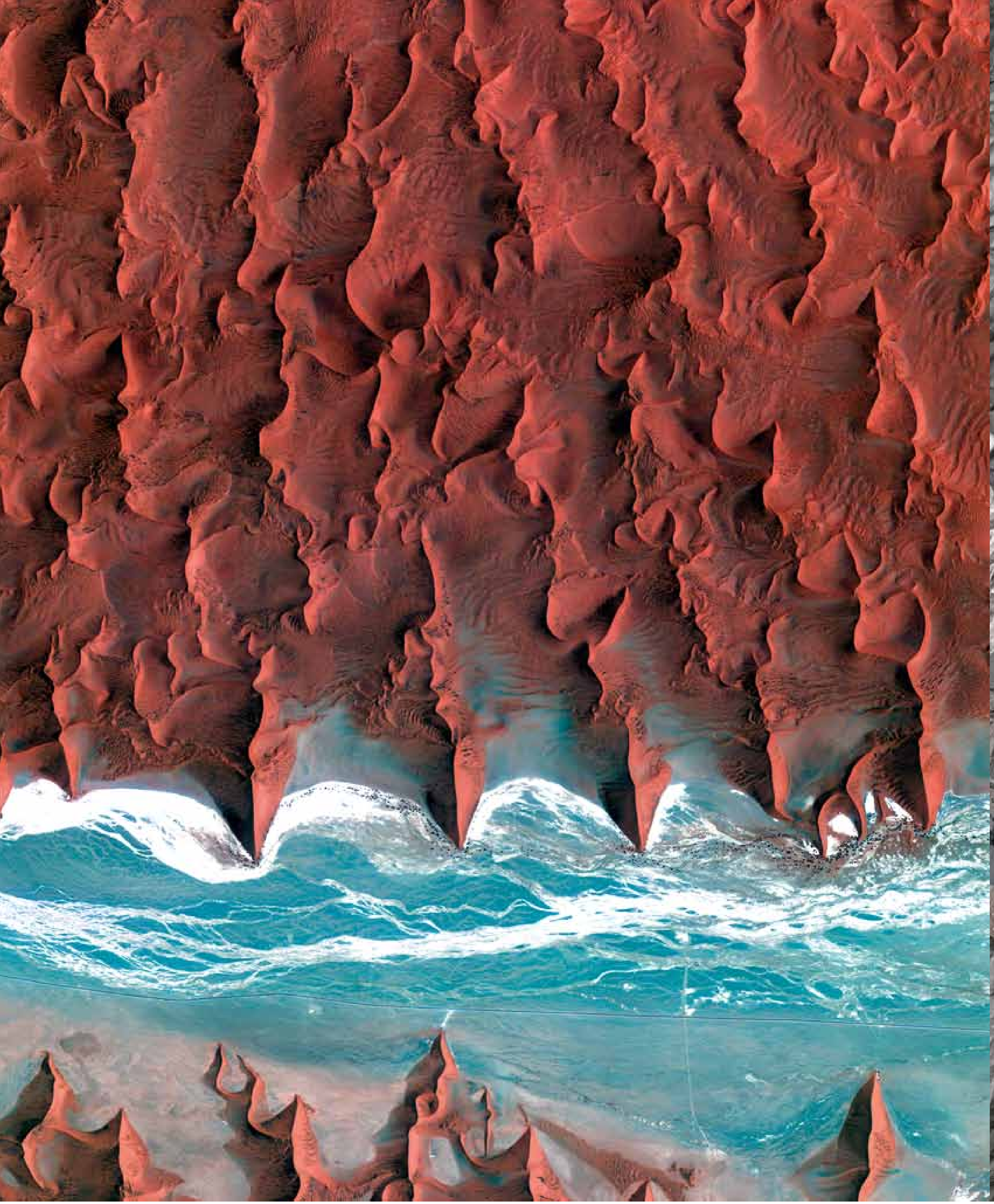
RECREATING NATURE'S ARTWORK

Japanese physicist Ukichiro Nakaya (*below*) has been credited as being the first person to create an artificial snowflake in the lab. Nakaya's obituary in the journal *Arctic* stated, "At the time Nakaya took over the new department of physics [in Japan's Hokkaido University], there was a minimum of equipment and few funds available for research. But he did have a microscope and an unlimited number of natural snow crystals during the long winters."

In total, Nakaya captured around 3,000 images of natural snow crystals, and later proposed a general classification system for snowflakes. Afterwards, he began trying to recreate nature's artwork in the laboratory.

In 1933, he succeeded, growing the world's first artificial snowflake on a single rabbit hair, by circulating water vapour in a cooled chamber. This discovery would allow scientists to study snowflakes more closely under controlled conditions, helping them to understand how the crystals form.

His 1954 book, *Snow Crystals: Natural and Artificial,* has been out of print for years, but as SnowCrystal.com (a website created by a professor of physics at the California Institute of of Technology, in the United States) puts it: "Nakaya's book offers a superb look at a scientific investigation which begins with almost nothing, and proceeds through systematic observation towards an accurate description of a fascinating natural phenomenon."



PHOTOS: KARI/ESA

SOUTH KOREA'S KOMPSAT-2 SATELLITE CAPTURED BOTH THIS IMAGE, FEATURING ROLLING HILLS OF FARMLAND IN THE US STATE OF WASHINGTON, AND THE IMAGE OPPOSITE, WHICH PICTURES THE SAND SEAS OF THE NAMIB DESERT IN SOUTH AFRICA. THE BLUE AND WHITE AREA IS THE DRY RIVER BED OF THE TSAUCHAB. BLACK DOTS OF VEGETATION ARE CONCENTRATED CLOSE TO THE RIVER'S MAIN ROUTE, WHILE SALT DEPOSITS APPEAR BRIGHT WHITE

WITH BOTH FRACTALS AND PRINTING TECHNOLOGY NOW GOING THREE-DIMENSIONAL, ONE DAY SOON YOU MIGHT BE ABLE TO PRINT EDUCATIONAL FOOD ITEMS, SUCH AS FRACTAL CEREAL, FOR YOUR CHILD'S BREAKFAST

PHOTO: JANNE KYTTANEN, CREATIVE DIRECTOR 3DSYSTEMS (MAIN)

Nature's functions, he says, rely on taking a simple pattern, and then repeating the pattern over and over again. "You need only two pieces of information — one that describes the basic pattern, and another that describes how to repeat it. This simple formula allows something as complicated as a tree or a cloud to be built," he explains. "Out of such complex, rough objects you get amazing functionality that smooth-shaped objects don't have."

"It is no coincidence that lungs and trees share a similar shape," adds Dr Adam Micolich, Taylor's colleague and an associate professor with the University of New South Wales in Australia. "They represent a way of getting a large surface area to exchange resources — in this case oxygen and carbon dioxide — with a small volume cost."

Micolich, who works on nanoelectronics, says this principle is being used to create ultra-efficient batteries that maximise the size of the anode/cathode interface without taking up a lot of space. Elsewhere, fractal antennas are widely used in mobile phones, to maximise the amount of surface area that can receive a signal. Chemical engineers too, are applying their understanding of the fractal nature of combustion processes to reduce the number of volatile gases released when burning coal. And in nanoelectronics, fractally branching patterns of energy dispersal have led to the design of highly efficient energy circuits for solar cells and nanoscale objects.

> "IF YOU WANT A BIG AREA TO TRANSFER LOTS OF STUFF AND YOU DON'T WANT TO CONTRIBUTE VOLUME BECAUSE OF COST, **USING A FRACTAL DESIGN IS A GREAT WAY TO ACHIEVE IT"**

"If you want a big area to transfer lots of stuff and you don't want to contribute volume because of cost, using a fractal design is a great way to achieve it," Micolich says.

DIGITAL MAGIC

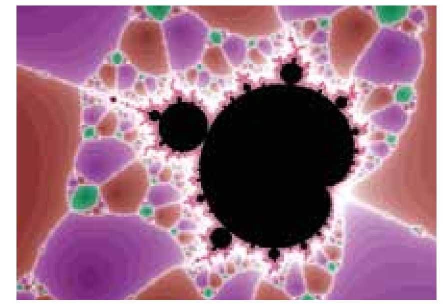
Fractals also hide in plain sight in movies as well, thanks primarily to Pixar Animation's co-founder and senior scientist Loren Carpenter. Using his knowledge of fractals to develop image synthesis, the fundamental technology that is behind digital film-making, Carpenter's groundbreaking fractals work would eventually go on to earn him an Academy Award.

FAMOUS FRACTALS

MANDELBROT SET

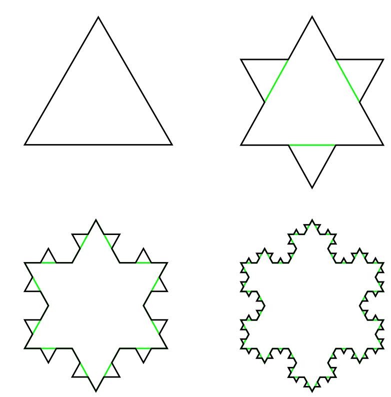
As well as measuring natural shapes, Mandelbrot is perhaps best known for popularising a shape that had never before been seen. He generated the eponymous pattern by feeding complex numbers into a very simple equation that he created, plotting the resulting number geometrically and then plugging the number back into the equation, and iterating the process thousands of times.

This 1979 discovery was only made possible because Mandelbrot had access to the processing power of computers at IBM, where he worked at the time. Since that time, images of the Mandelbrot set have appeared everywhere, from clothing and car stickers to Cambridge crop circles.



THE KOCH SNOWFLAKE

First described in 1904 by Swedish mathematician Niels Fabian Helge von Koch, the shape and complexity of the Koch snowflake allegedly inspired Mandelbrot in his quest to measure the British coastline. Using an equilateral triangle as its base, each side of the triangle has another triangular kink added in its centre, with each side of the triangular kink equivalent to one-third of the length of the original side. Kinks can be added at smaller and smaller scales on each side of the triangle to create a snowflake pattern.



"I started out as a computer programmer back in the '60s when computers were hard to come by," he explains, adding that while studying computer science he worked at Boeing on rendering algorithms that showed how concept aircraft might fly.

"Everything Boeing puts out has a mountain in the background. It implies the aircraft is safe because it is flying over mountainous terrain," he says. To display the concept aircraft in classic Boeing style, he wanted to make pictures of aeroplanes passing in front of mountains.

"Then in 1979 I found a book by Benoit Mandelbrot, describing the math behind fractals and containing some intriguing pictures of fractally-generated landscapes," he recalls. "I chewed over this for a couple of months and came up with a family of algorithms to generate fractals to create natural-looking animated landscapes."

Natural landscapes have a lot of randomness, he explains. If you make an image of a static object it has no randomness, so to make it appear realistic you need to add randomness.

"Computers can generate random numbers by scrambling a set of numbers and then giving them back. You can do this repeatedly and use these numbers to plot points on a plane (surface) that will defy predictability and create a picture," he says. "But when you move the camera, or the image is viewed from another angle it doesn't make sense, because when you said you wanted random, you got random."

When creating artificial landscapes or other natural-looking images, consistent randomness is required, Carpenter adds. "To create a natural-looking image you zoom in very closely on an image and then add new bits of randomness in the cracks in the image using fractal geometry."

Using his algorithm, Carpenter made a film, *Vol Libre,* showing a flyover of a mountain range that appeared remarkably realistic compared with the kinds of animation available then. To put it in perspective, *Pac-Man,* the gold standard animated video game of its time, didn't come out until a year later.

"It was exciting because I figured out how to make movie-quality images at a time when animation was limited to crude video game images," he says. Shortly afterwards he was hired by Lucasfilm to establish a computer division that would complement their special effects business.

The first major project for the fractal-based image rendering software he created was for the movie *Star Trek II: The Wrath of Khan,* where Carpenter and his team created an entire planet. "In the movie, the Genesis device was intended to convert dead planets into living ones. We wanted to realistically simulate this and not have people focusing on bad special effects."

To create the planet, they generated a fractal landscape and fly-by paths that allowed the image to remain realistic even as the viewpoint changed.

"The film-makers were so happy with the result that they used the sequence in the next two or three movies," he laughs.

Now, Carpenter says, there are lots of fractals in movies. They are applied to curved surfaces as a way to make rocks or pebbles in a stream more interesting. They're used to create reflectivity, bark on trees, and dirt on the ground. Simulated waves use fractal systems, while lighting programs simulate reflections off wrinkled surfaces. Some software specialises in smoke production, others in making water move realistically.

But while extraordinarily realistic animation can be achieved thanks to the use of fractals, generating realistic-

THE ABILITY TO DETECT PEOPLE THAT AREN'T RIGHT IS AN EVOLUTIONARY THING, KNOWN IN ANIMATION AS THE "UNCANNY VALLEY"

looking people that don't have the audience squirming in their seats is another matter.

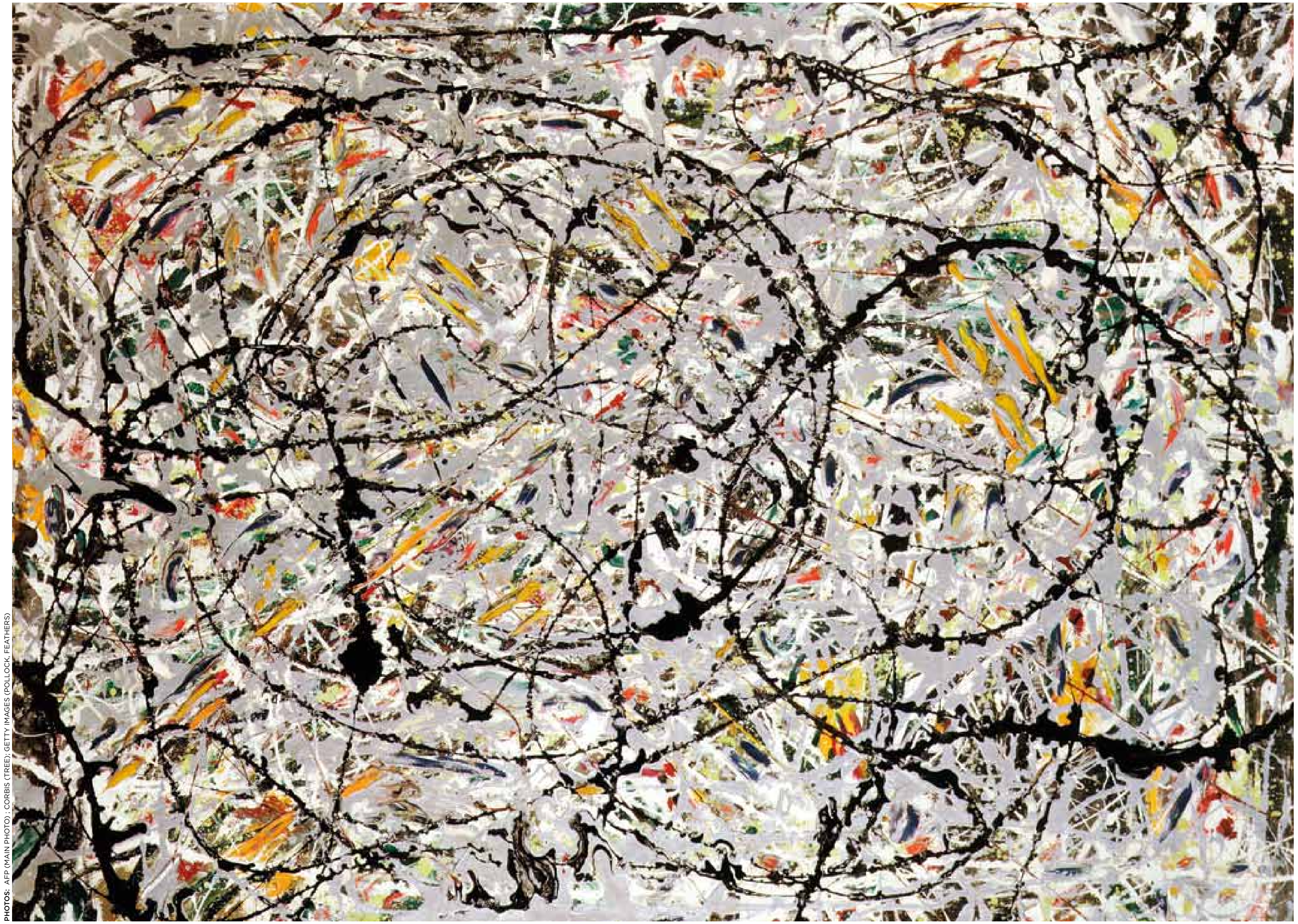
"At Pixar, where up to a million hours are spent on each film, our dinosaurs or imaginary landscapes are rendered so that people accept them as a valid world for the characters," he says. "However, if you have a cartoon human, people are quite happy to buy the caricature, so we make the skin of our characters look plastic."

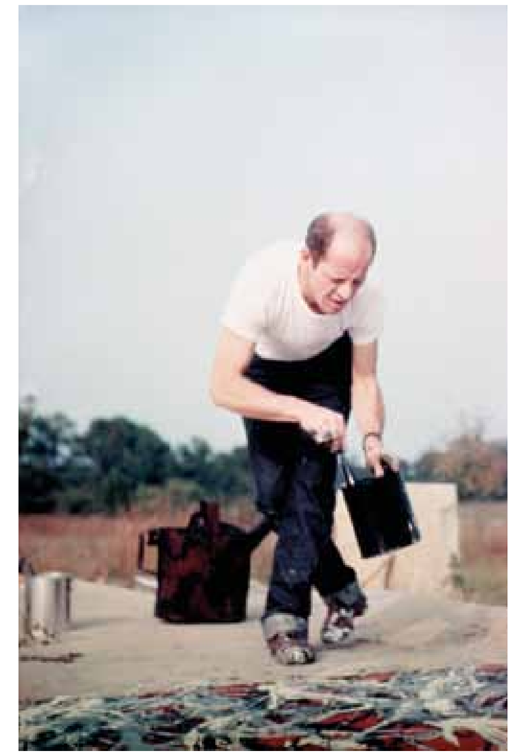
But to make a realistic-looking human is much harder, he says. "It can be achieved at some scales, such as when the person is only a quarter the height of a screen, but when it comes to facial or hand close-ups, not everyone buys it and it becomes creepy — and the last thing we want is for viewers to move between happy and creepy."

He notes that the human ability to detect people that aren't right is an evolutionary thing, known in the animation business as the "uncanny valley". "The term was coined by Japanese robotics professor Masahiro Mori in 1970, along with a theory that says when human features look and move almost, but not exactly, like natural human beings, it causes human observers to feel revulsion." The "valley" refers to the dip in a graph of the comfort level of people viewing the not-quite-human features.

"The human visual system is drawn to natural-looking objects, and the eye is a natural and powerful fractal detector, much more powerful than a computer," says

PHOTOS: AFP (MAIN PHOTO); CORBIS (TREE); GETTY IMAGES (POLLOCK, FEATHERS)





ABOVE: JACKSON POLLOCK'S *UNDULATING PATHS*, MADE IN 1947. TAYLOR SAYS POLLOCK (*ABOVE, RIGHT*) DELIBERATELY IMBUED HIS ARTWORK WITH A FRACTAL QUALITY **LEFT:** A BLACK OAK TREE IN THE YOSEMITE VALLEY IN THE UNITED STATES, WITH A GRANITE WALL AS A BACKDROP. THE FRACTAL PATTERN IS PLAINLY VISIBLE **RIGHT:** THE ELABORATE FEATHERS OF A MALE PEACOCK'S TAIL ARE A STUNNING EXAMPLE OF FRACTALS IN NATURE

Taylor. "At an unconscious level, our bodies are trained to respond to fractal patterns, which we find soothing."

FRACTAL DETECTION

So if fractals are everywhere, why did it take so long to for them to be recognised? "It's one of those big embarrassments," he laughs. "In the mid-1800s mathematicians started to generate patterns based on layering. They called them 'pathologicals' and thought their patterns superior to anything found in nature, which they thought was generated by random, haphazard processes."

Taylor notes, "It's a matter of conditioning to what we consider to be shapes. We are brought up both in maths and art to recognise patterns that use Euclidean geometry." That is to say, shapes like circles, squares, spheres and cubes.

"We are not consciously trained to look at messes and see patterns," he adds. "One of the huge contributions of fractal geometry is that it has made us reassess exactly what is a pattern, and has given us a language to understand patterns in nature."

As to why we missed fractals, Micolich feels this is understandable. "How did we miss gravity when it was right in front of our face?" he asks. "People finally got around to studying these natural features, and Mandelbrot pulled it all together."

While Micolich currently works on nanoscale electronic devices, his discovery of the way electric current travels fractally in nanodevices (it branches like a river

system) later resulted in unmasking a hoard of fake Jackson Pollock artworks.

Taylor was Micolich's PhD supervisor back in the 1990s. By day they worked on nanoelectronics and by night they analysed the compelling patterns and organic shapes of Jackson Pollock's work, a source of fascination to Taylor since he was a child.

"There is a strong relationship between maths and art." After completing a PhD in physics, Taylor went to art school, where he pondered the works of "Jack the Dripper".

"I knew about fractals from the study of physics, and once back at art school thought a lot about Pollock. The defining moment came when I built a pendulum structure filled with paint that was blown about by the wind, which has its own fractal motion."

"When the artwork was finished, I noticed it looked a lot like a Pollock painting, so once Adam [Micolich] came up with a technique to analyse fractal patterns, we decided to put it to the test by applying it to Pollock's *Blue Poles*."

They found that not only is there an underlying fractal character to Pollock's work, the artist knew exactly what he was doing when he created them. "Pollock's natural style of painting repeats general qualities, rather than the intricate iterations that characterise fractal imagery like the Mandelbrot set," says Taylor.

"He may have started pouring with different intentions, and he may not have understood the patterns, but he understood their quality," he continues. "Pollock spent 10 years refining his technique, and docked his canvases to focus on the central, most fractal part of the work — the fractal quality of the paintings deteriorates towards the edge of the paintings."

Taylor's expertise on Pollock was later called upon in what became known as the "Matter Matter", one of the longest running and most contentious art controversies of the 2000s. A treasure trove of supposed Pollock paintings were found in a storage locker by a man named Alex Matter, but their authenticity was almost immediately called into question. Taylor used the Pollock pattern analysis methodology he had created with Micolich to determine whether the newly discovered paintings were consistent with the style the artist used in his other works.

Unpopularly, Taylor's analysis found significant differences between those paintings and established Pollocks. "This was bad news for a lot of people, and controversy over the findings raged for around a year until a group of scientists from Harvard showed that some of the pigment used in the paintings dated from the 1980s. Pollock himself died in 1956, which put the matter to rest."

But just as this scandal was finally going away, another Pollock scandal was arising, with a famous private gallery in New York's collection of Pollock artworks being found to have been painted by a Chinese artist.

"Pollock is a double-edged sword," Taylor says. "At a superficial level people are convinced that they can paint in his style, but there is an amazing equilibrium and composition that can't be replicated, and fractals play a key role in creating this."

He adds, "These events show that there is a bright future for computer scientists working in the area of art authenticity. A scholar might know whether a work is authentic or not, but without quantitative evidence might not be able to convince a jury. But while it will never replace scholarship, science, and in this case fractal analysis, can quantify the materials and processes artists use to create their work — and help ensure that the truth comes to light."

THIS LANDSAT IMAGE SHOWS THE MISSISSIPPI RIVER DELTA, IN THE UNITED STATES. THE WAY THE RIVER CUTS THROUGH LAND TO REACH THE SEA LOOKS VERY SIMILAR TO THE SPREAD OF A TREE'S BRANCHES OR THE SHAPE OF A LIGHTNING BOLT
LEFT: ROMANESCO CAULIFLOWERS, WHICH HAVE A NATURALLY FRACTAL SHAPE, LOOK ALMOST TOO PRETTY TO EAT

PHOTOS: USGS/ESA (MAIN); GETTY IMAGES (ROMANESCO CAULIFLOWER)