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Navigation performance in virtual environments varies with fractal dimension of landscape

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ABSTRACT

Fractal geometry has been used to describe natural and built environments, but has yet to be studied in navigational research. In order to establish a relationship between the fractal dimension (D) of a natural environment and humans' ability to navigate such spaces, we conducted two experiments using virtual environments that simulate the fractal properties of nature. In Experiment 1, participants completed a goal-driven search task either with or without a map in landscapes that varied in D. In Experiment 2, participants completed a map-reading and location-judgment task in separate sets of fractal landscapes. In both experiments, task performance was highest at the low-to-mid range of D, which was previously reported as most preferred and discriminable in studies of fractal aesthetics and discrimination, respectively, supporting a theory of visual fluency. The applicability of these findings to architecture, urban planning and the general design of constructed spaces is discussed.

1. Introduction

Human performance in complex virtual environments has often been studied using regular geometric structures such as mazes (e.g., Chrastil & Warren, 2013; Moffat, Hampson, & Hatzipantelis, 1998; Wolbers & Büchel, 2005). While such paradigms allow for precise experimental control, they do not capture the complexity and roughness inherent in natural environments. In contrast, work that has been conducted to explicitly capture features of natural environments (e.g., Darken & Banker, 1998; Witmer, Bailey, & Knerr, 1995) often involves a meticulous recreation of a specific physical space, which can be both time consuming and prohibitively expensive due to the resources required to accurately survey a large natural space. Stürzl, Grixa, Mair, Narendra, and Zeil (2015) propose the use of laser-based environmental recreation for modeling natural environments. Such an approach provides high quality recreation with less time cost. However, it still requires the physical collection of environmental information and is expensive to carry out.

Given this variety of approaches to modeling environments, there has been no clear consensus on how best to measure and manipulate environment complexity in a generalizable way when assessing navigation performance. O'Neill (1992) and Stankiewicz, Legge, and Schlicht (2001), for example, operationalized complexity as the number of corridors within a virtual building. While useful for some environments, such a metric has little generalizability to natural spaces. This trend towards either geometric simplification or effortful recreation of the environment in navigation research can be explained by the fact that spatial environments characterized by Euclidian geometry, such as mazes composed from straight lines and angles, are relatively simple to model using modern computer software. The problem with this approach is that Euclidian features do not accurately characterize many natural phenomena.

Instead, natural environments often display complex patterns that are irregular and repeat at increasingly fine size scales, and are best described using fractal geometry (Bolliger, Sprott, & Mladenoff, 2003; Mandelbrot, 1982). In this work we employ the fractal dimension (D) as a generalizable means of manipulating and describing the complexity of a virtual environment. This approach was adopted by Voss (1988) who used fractal properties to generate virtual simulations of natural environments, such as moons and

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planets. However, given the limitations of computing resources at the time, the landscapes were static and embedded in a two dimensional plane, and could not therefore be navigated. Currently available graphical resources now make it feasible to render fractal landscapes in virtual environments in real-time.

In order to understand D, consider the boundary edge of a natural object such as an island's coastline. D quantifies the relative amounts of coarse and fine structure present in the coastline, and can range between traditional Euclidian dimensions – from $D = 1.0$ of a smooth line with no fine structure (lowest complexity) to $D = 2.0$ with a large amount of fine structure (highest complexity) (Mandelbrot, 1967). The coastline of Australia, for example, has a relatively low D (Gouyet & Mandelbrot, 1996; Richardson, 1961). In contrast, the coastline of Norway is much more finely structured with a myriad of fjords defining it, making it much higher in D (Gouyet & Mandelbrot, 1996; Richardson, 1961). Methods developed to measure D have been used to characterize diverse natural environments (Bolliger et al., 2003), animal habitation patterns (Palmer, 1992), and urban cityscapes (Encarnação, Gaudiano, Santos, Tenedório, & Pacheco, 2012).

While D has yet to be used to characterize environments in virtual navigation tasks, its impact has been studied in a number of other perceptual contexts. These studies have investigated the perception of computer generated images (Aks & Sprott, 1996; Spehar & Taylor, 2013), natural environments (Hagerhall, Purcell, & Taylor, 2004), and works of art (Taylor, 2006; Taylor, Micolich, & Jonas, 1999). Spehar, Clifford, Newell, and Taylor (2003), for example, explored the relationship between D of a series of images (including natural, mathematical, and human-generated fractals) and aesthetic judgment. Studies such as theirs and others point to low-to-mid range D stimuli, typically with the range of $D = 1.3–1.5$, as being the most preferred by individuals (for a review see Taylor, Spehar, Van Donkelaar, & Hagerhall, 2011). Taylor et al. (2011) discuss a resonance theory of aesthetics in which eye movements, which follow mid-range fractal trajectories, may resonate with the inherent fractal structure of natural patterns. Aks and Sprott (1996) propose an exposure model with preference for low-D fractal values due to exposure to nature's fractal patterns. This and other research also supports a processing fluency model in which the visual system processes low-to-mid D fractals with relative ease, resulting in a heightened aesthetic experience.

Processing fluency might also enhance capabilities such as the ability to detect and discriminate fractals (Spehar et al., 2015), to maintain attention when observing fractals (Hagerhall et al., 2008, 2015), and also to heighten pattern recognition skills. In terms of pattern recognition, research reveals that fractal images of low-to-mid D preferentially activate the object perception and recognition areas of the visual cortex (Bies, Wekselblatt, Boydston, Taylor, & Sereno, 2015) and allow for a larger number of percepts to be formed (Bies et al., 2016). This is consistent with earlier behavioral studies in which the capacity to perceive shapes in ambiguous fractal images was shown to peak in the low D range (Rogowitz & Voss, 1990). In this paper, we test a visual processing fluency model in which we hypothesize that there may be optimization of visual-spatial information processing in the low-to-mid D range for the purpose of goal-directed navigation.

The tasks in this study utilize virtual environments presented on a screen as a proxy for physical environmental navigation. Virtual environments have been used in navigational research for several decades (for reviews, see Loomis, Blascovich, & Beall, 1999; Nash, Edwards, Thompson, & Barfield, 2000), and have been shown to be reasonable approximations of physical environments for transferring navigational skills to their real-world equivalents (Arthur & Hancock, 2001; Richardson, Montello, & Hegarty, 1999). The use of virtual environments in the following experiments makes possible the fine experimental control over landscape topography that would be otherwise impossible in a physical context where participants may be asked to explore an environment.

Much work on virtual navigation has focused on the utilization of landmarks as a means for an individual to orient and guide themselves. A basic distinction in these studies is between local vs. global landmarks, with local landmarks associated with smaller-scale information and global more distant landmarks, larger-scale information. We frame the discussion of this work in a novel way by considering fractal landscapes which, by definition, contain landmarks defined by particular mixes of large-to-small scale structure. Steck and Mallot (2000), for example, distinguish between global landmarks as those that can be used for orientation at any location, and local landmarks as those that can be used to guide more fine-scale navigational strategies. Whereas these categories of landmarks might normally be set by the designer of a virtual environment, in an environment generated using the fractal properties of nature, landmarks are understood to be any feature of the generated landscape which may have characteristics that can be utilized for the purpose of successful navigation, such as those outlined by Vinson (1999). The global-local distinction is thus one that may come about as a result of the environment's fractal properties. Wiener and Mallot, (2003) suggest spatial navigation takes place by making use of a fine-to-coarse planning heuristic when obtaining landmark information from the environment. This involves the simultaneous use of local fine environmental detail and coarse distant detail in planning one's path through an environment. Different levels of coarse and fine structure are inherent in fractal patterns of varying D, and may allow for individuals to perform best when the environment contains structure that can be taken advantage of by the planning heuristic. By associating environmental topographies of particular D with navigational performance within these environments, we hope to discover the ratio of coarse-to-fine structure which may be best utilized during route planning.

Given the use of D in characterizing natural environments as well as the links found between D and human perception, fractal dimension may serve a useful role in creating and manipulating environmental complexity for research using virtual environments. In the following experiments we present D as a systematic and generalizable means of manipulating the complexity of a spatial environment, and examine the effect of this complexity on navigation performance. We predict that performance will be highest within the low-to-mid range of D found in other perceptual contexts as suggested by a processing fluency model.

2. General methods and materials

2.1. Stimuli

The environment topographies were generated using an inverse Fourier method similar to that described by Spehar and Taylor (2013). The algorithm summed a set of cosine waves with spatial frequencies f and amplitudes defined by the spectral slope α given by the formula $Z = f^{-\alpha}$. This procedure created a landscape pattern in which large scale low frequency waves had a large amplitude, while smaller scale higher frequency waves had a lower amplitude. Furthermore, the range of chosen frequencies generated a landscape that scaled across two orders of magnitude, which is representative of the extent to which many natural environments show fractal properties at varying scales (Balboa & Grzywacz, 2003; Koch, Denzler, & Redies, 2010). Using numbers generated from a Gaussian distribution, the phases of the waves were then randomized to generate the statistical scale invariance of natural scenes. For each of 5 phase conditions, 5 values of α were used to generate a total of 25 landscapes. Each landscape was interpolated to generate a surface with pixel counts of 1024 $\times$ 1024 in the x and y directions (See Fig. 1.1). The D values of their profiles were 2.1, 2.3, 2.5, 2.7, 2.9 and these values were verified using a fractal box-counting analysis (Fairbanks & Taylor, 2011).

For each image, the median greyscale intensity was assessed and pixels with intensities below the median were set to 0 intensity. This was done in order to create the appearance of a flat ground with various features and landmarks protruding (See Fig. 1.2). The topology of the landscapes retained the fractal dimension, with negligible measured variation from the original generating parameter. When the edges of the topographic features were measured using box counting, we found their D values to be 1.1, 1.3, 1.5, 1.7, 1.9. It is these values which we shall refer to when discussing the D of the island set going forward. When the length of these feature edges were measured, we found a strong linear relationship between dimension and edge length, $r(25) = 0.96$, $p < 0.001$. Additionally, a flat topography was generated and designated as dimension 1.0, and had edge lengths of zero. This flat topography was duplicated five times, to match the number of terrains at all other D levels. These D values were used as independent variables in the experiments that followed. The landscapes were then circumscribed, to give the appearance of circular islands (See Fig. 1.3), and so are referred to as landscapes or islands throughout the text. As in the Morris water maze, this circumscribing was done in order to prevent the use of corners in the landscape as global landmarks (Morris, 1984). Chai and Jacobs (2009) employ a similar strategy in the design of their tasks exploring cue use in navigation. The generation process is illustrated in Fig. 1. Additionally, an example of the island features at each level of D is provided in Fig. 2.

2.2. Experimental setup

The landscapes were presented as virtual island environments created using Unity3D game engine software on a 2008 Mac Pro with a 30″ screen, and rendered at a resolution of 2133 $\times$ 1600 pixels. Participants were seated two feet away from the display, which comprised 60 degrees of visual angle. Water was rendered outside the virtual island terrains, and participants were unable to move beyond these circular boundaries. Participants navigated an avatar through the confines of the circular environments from a first-person perspective using a PlayStation 3 controller. The virtual island environments were rendered at a scale of 200 m in diameter, with the maximum height of the landscape set to 50 m. Participants could move their avatar around the flat surface of each island, as well as over terrain of an incline less than 45°. Elevated terrain prevented participants from seeing from one end of an island to the other, except for the D = 1.0 islands. In some experimental conditions, a topographic map of the island was displayed for use in

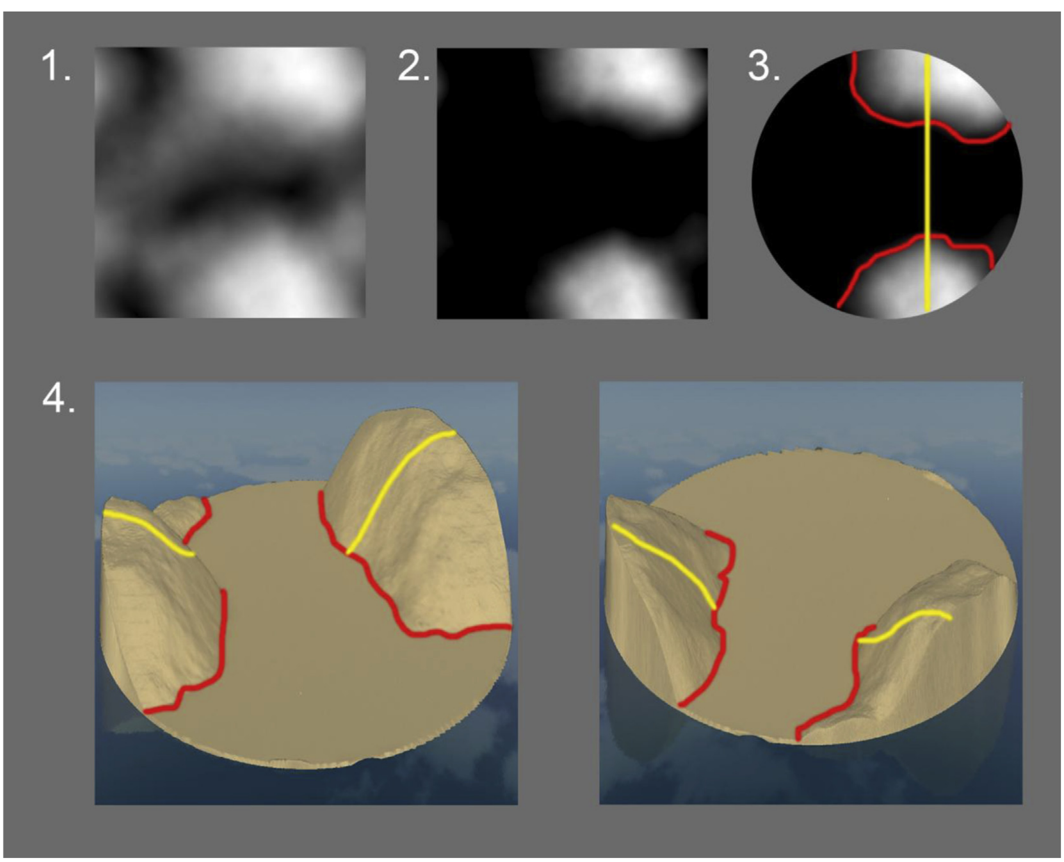

Fig. 1. Illustrations of the topography generation process for a terrain with edges of D = 1.1. 1. The greyscale output terrain from the inverse Fourier generation process. 2. The terrain after having intensities below median intensity set to zero. 3. The terrain after having been circumscribed. 4. View of the terrain from two angles as rendered in the virtual environment. Feature edges are highlighted in red to indicate the aspect of the environment which is reflected in the reported fractal dimensions (D). For reference, an example feature profile is highlighted in yellow to indicate profile fractal dimension. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

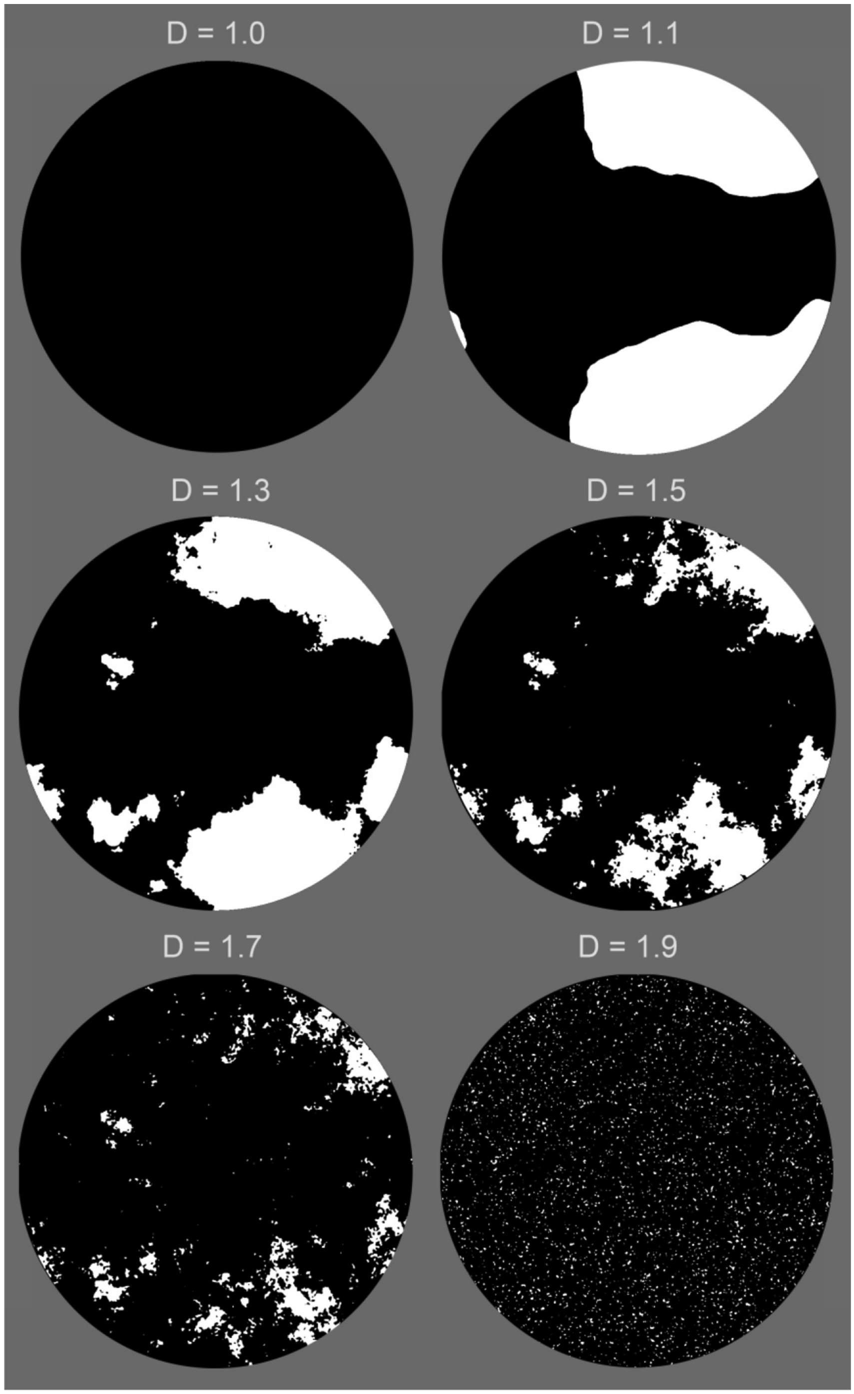

Fig. 2. The topography of a set of generated landscapes from one of the five phase maps utilized in this study. White designates landmarks, and black designates ground. D is indicated above each of the 6 landscapes: 1.0 (top-left), 1.1 (top-right), 1.3 (mid-left), 1.5 (mid-right), 1.7 (bottom-left), and 1.9 (bottom-right).

navigation. Four examples of a participant's view from the displayed environment are presented in Fig. 3.

2.3. Analysis

All behavioral data was collected automatically within the experiment's software, and analyzed using the JMP 12 Statistical Package. Demographic data was additionally collected from a post-experiment survey.

3. Experiment 1: search tasks

3.1. Purpose

Our two stated goals above were to establish the D value of a landscape as a viable measure of environmental complexity, and to test a visual fluency model in which low-to-mid D fractal landscapes are most successfully processed for the purpose of goal-directed navigation. Given that D has not previously been used as a means of manipulating environmental complexity in navigation

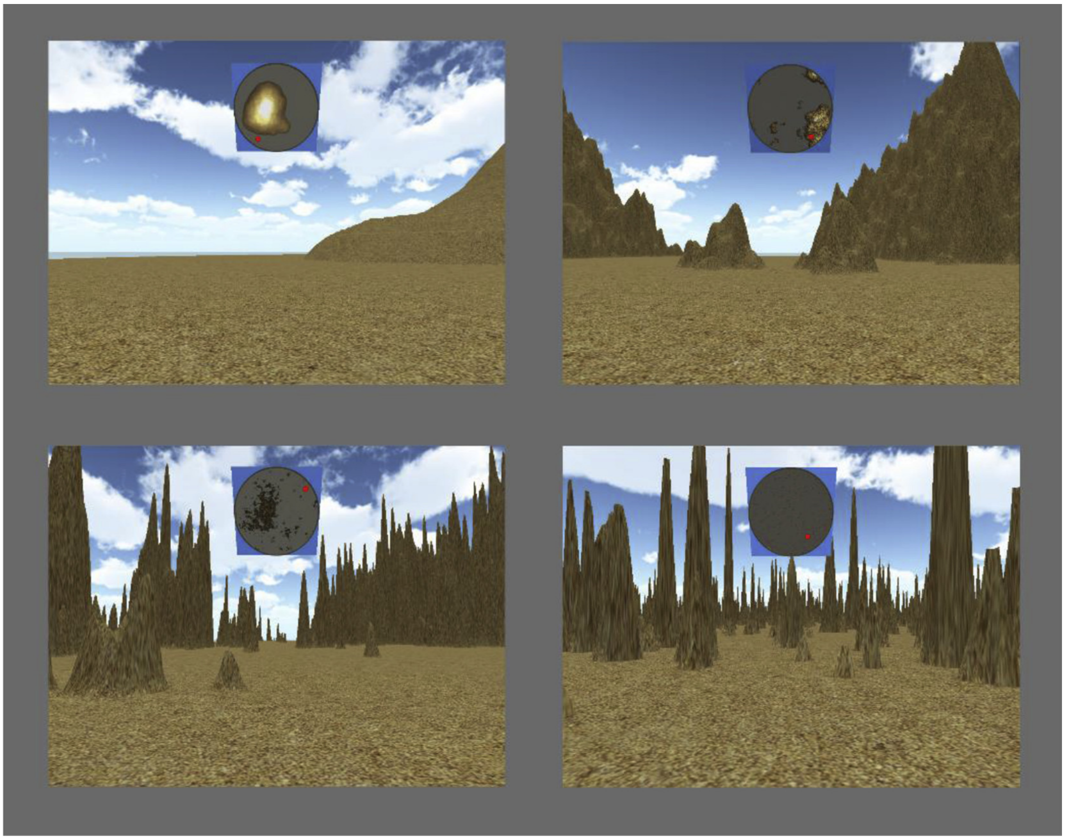

Fig. 3. Examples of first-person perspective views in Condition 2 (Map). The D of the landscapes are 1.1 (top-left), 1.3 (top-right), 1.7 (bottom-left), and 1.9 (bottom-right). The displayed map is shown to participants in certain experimental conditions described below. The red dot on the map indicates the position of a target object. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

tasks, we first conducted a simple navigation experiment under various conditions to begin answering these research questions. In Experiment 1, participants were instructed to explore landscapes composed of a range of fractal dimensions, and complete a goal-driven search task either with or without a map and/or a distractor object. Completion speeds (all Conditions) and accuracy (Condition 3) were collected. In Condition 1, participants engaged in a goal-directed search task in which they were instructed to find a visible goal within the environment without the use of a map. By doing so, we were able to establish the effect of D as a measure of complexity on performance when engaged in goal-directed search.

We additionally explored the effect of a topographic map on performance in this task with the goal location marked on the map, and without (Condition 2) or with (Condition 3) an additional distractor object (See Fig. 3). The use of topographic maps in navigation has been studied previously (e.g., Malinowski & Gillespie, 2001; Montello, Sullivan, & Pick, 1994; Tkacz, 1998) but never before via dynamic exploration in simulated natural environments. The purpose of these map task manipulations was to gain an understanding of the extent to which fractal information present in a topographic map can be interpreted and utilized when navigating a fractal environment. Specifically, in these two map conditions participants used the map to help determine where they were on the island and help guide their way to the marked goal location. To do this successfully, participants must: 1) identify and match landmarks in the environment with their representations on the map, 2) use these landmarks to determine their own position (self-location) and orientation in the environment and relative to the goal, and 3) use this position and orientation knowledge to plan a route to the goal. The purpose of including a distractor object in Condition 3 was to ensure that participants used the map to successfully navigate to the target (rather than distractor) object location.

Finally, we examined navigational performance in fractal environments when participants were given information displayed on a map relaying both self and goal location in real-time (Condition 4). By eliminating the need for active search or landmark interpretation, this final task served as a baseline condition which was used to measure performance relative to the other three conditions. For a detailed description of these conditions, see Section 3.2.2.

3.2. Method

3.2.1. Participants

Seventy-four (30 Male) participants were recruited from the University of Oregon undergraduate research pool to participate in this experiment, and were provided with credit in a psychology course for doing so. No recruited individuals had participated in previous similar studies, and demographic information concerning major was not collected. This study was carried out in accordance with a protocol approved by the Research Compliance Services of the University of Oregon. All participants provided informed consent before undertaking the experiment. Ages of participants ranged from 18 to 34 years (*Mdn* = *19*). Sixty-one of the participants were randomly assigned to one of four conditions, with an additional 13 assigned to Condition 3 due to the additional power required to examine a second dependent measure. The demographics of these participants did not differ from that of the others. Condition participation was as follows: 15 in Condition 1; 15 in Condition 2; 28 in Condition 3; and 16 in Condition 4.

3.2.2. Procedure

Participants were instructed to explore each island in the virtual environment and search as quickly as possible for a coconut (goal) randomly placed within the outer two-thirds of the island. Goals were never placed within the inner third of the environment in order to increase task difficulty in map-present conditions. Both the participant's starting position and the goal were always located on the flat ground of the landscape. To complete each trial, participants were instructed to use the PlayStation controller to navigate the environment, and move to the position of the goal. The controller allowed participants to move in all directions within the confines of the terrain using the left analogue stick, and to adjust the angle of view from their location using the right analogue stick. Upon arrival at the goal, the trial ended, and the participant was immediately taken to the next trial. If a participant failed to find the coconut within 180 s, they were automatically taken to the next trial. This cutoff time was chosen as an upper limit in order to ensure that participants would be able to complete the required number of trials within the 60 min of the experiment while also minimizing the likelihood of prematurely ending a trial that a participant is likely to successfully complete.

The number of trials varied by condition (see Table 1). The presentation sequence of islands and the goal position on each island were randomized for every participant. The design of the experiment was between-subjects, with each participant completing the procedure for only a single condition. In all conditions, the first trial was used for training, in which the experimenter coached participants as they completed the task to ensure all participants understood the task. The specific manipulations for each condition are described in Table 1.

In Table 1, the column labeled "Map with goal?" indicates whether there was an always-present map of the topography of the landscape. This map additionally marked the position of the goal with a red circle, and was aligned with respect to the landscape, rather than with respect to the heading direction of the participant. "Distractor on island?" indicates whether there was also a second coconut randomly placed on the ground of each island which served as a distractor. The location of this distractor was not marked on the map. Participants were informed that there was a distractor object in the environment that was not indicated on the map, and was to be avoided while completing the task. Unlike the goal position, a trial did not end upon arriving at the distractor position. However, the occurrence of a participant mistaking a distractor for the target was recorded as an error. In such cases, participants were instructed to continue searching for the true goal by the experimenter. "Map with self?" indicates whether the map also marked the position of the participant on the island in real-time. "Number of Trials" indicates the number of trials a participant was instructed to complete in each condition. The dependent measures consisted of time-to-goal, which was measured as number of seconds from the start of the trial until the goal was found, and error rate, which was the proportion of trials in which a participant mistook the distractor for the goal. Short clips of each condition are presented in Video 1.

Supplementary video related to this article can be found at http://dx.doi.org/10.1016/j.jenvp.2016.05.011.

3.3. Results

We examined the between-group variance in time-to-goal by condition with Levene's test, which found that the conditions had unequal variances; $F(3, 70) = 3.75$, $p = 0.003$. Welch's ANOVA was then conducted in order to remain robust to these unequal variances, as well as unequal condition sizes. The Welch's ANOVA compared the effect of condition type on time-to-goal, revealing significant differences in time-to-goal among the four conditions, $F(3, 70) = 200.81$, $p < 0.001$. A series of a priori Bonferroni corrected contrasts was undertaken to explore the nature of the differences between these groups. In order to establish the non-trivial nature of the experimental conditions, the first contrast compared the baseline condition (C4) with the three experimental conditions (C1, C2, C3). We found a significant difference between the experimental and baseline conditions, with Condition 4 ($M = 14.11$, $SD = 3.22$) having a much shorter time-to-goal; $t = 15.24$, $p < 0.001$. We then conducted a contrast to compare the map (C2 and C3) to the no-map (C1) experimental conditions. In doing so, we found that Condition 1 (No-map) ($M = 53.24$, $SD = 10.97$) had a significantly longer mean time-to-goal than Conditions 2 (Map) ($M = 44.46$, $SD = 7.6$) and 3 (Map and distractor) ($M = 48.87$, $SD = 8.21$); $t = 2.7$, $p = 0.008$. This demonstrates that participants with access to a topographical map of the environment and goal location exhibited significantly better performance when compared to participants without such a map. A final contrast was conducted to explore the effect of distractor targets (C3) within the map-present experimental conditions (C2 and C3). When comparing map-present Conditions 2 (Map) and 3 (Map and distractor), we found a trending, but non-significant difference between the two conditions; $t = -1.72$, $p = 0.088$. This suggests that participants using a map were likely not especially slowed by the presence of a distractor. These contrasts are presented in Fig. 4.

With an understanding of between-condition differences, we then analyzed the relationship between fractal topography and time-to-goal within conditions. Here we found an exponential relationship between time-to-goal and D within each condition, suggesting a linear increase in time-to-goal as D increases when logarithmically transformed. A simple linear regression was calculated to predict the logarithm of time-to-goal based on D for each of the 4 conditions. There were significant positive linear trends in the relationship between the D of the topography and the logarithm of time-to-goal for all three experimental conditions: Condition 1 (No-map), $F(1,88) = 231.08$, $p < 0.001$, $R^2 = 0.72$; Condition 2 (Map), $F(1,88) = 446.29$, $p < 0.001$, $R^2 = 0.83$; and Condition 3 (Map and distractor), $F(1,166) = 612.64$, $p < 0.001$, $R^2 = 0.78$. In baseline Condition 4 (Map w/self) we found a weaker but significant linear relationship between the two variables in question, $F(1,94) = 43.35$, $p < 0.001$, $R^2 = 0.34$. These trends are plotted in Fig. 5.

Table 1
Properties of the four conditions of Experiment 1.

Condition	Map with goal?	Distractor on island?	Map with self?	Number of trials	Dependent measures
1. No-map (Experimental)	No	No	No	30	Time to goal
2. Map (Experimental)	Yes	No	No	60	Time to goal
3. Map and distractor (Experimental)	Yes	Yes	No	60	Time to goal; Error rate
4. Map w/self (Baseline)	Yes	No	Yes	60	Time to goal

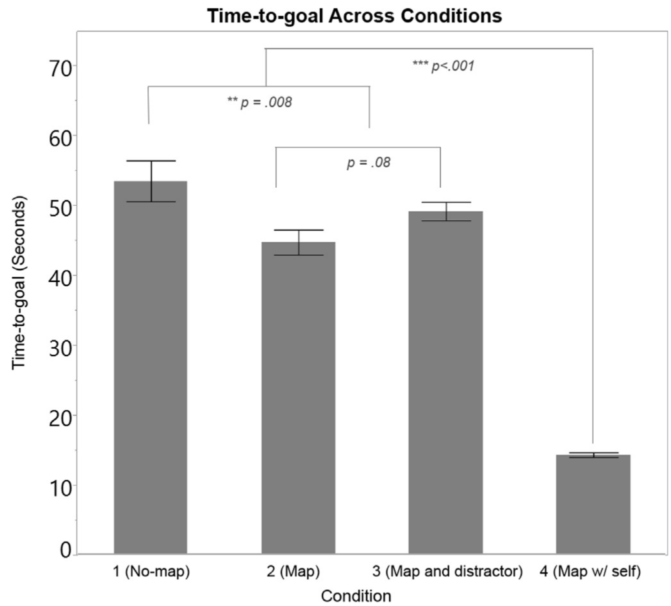

Fig. 4. The mean time-to-goal in seconds for each condition. Error bars represent standard error. Significance levels are indicated by "*" ($p \le 0.05$), "**" ($p \le 0.01$), and "***" ($p \le 0.001$).

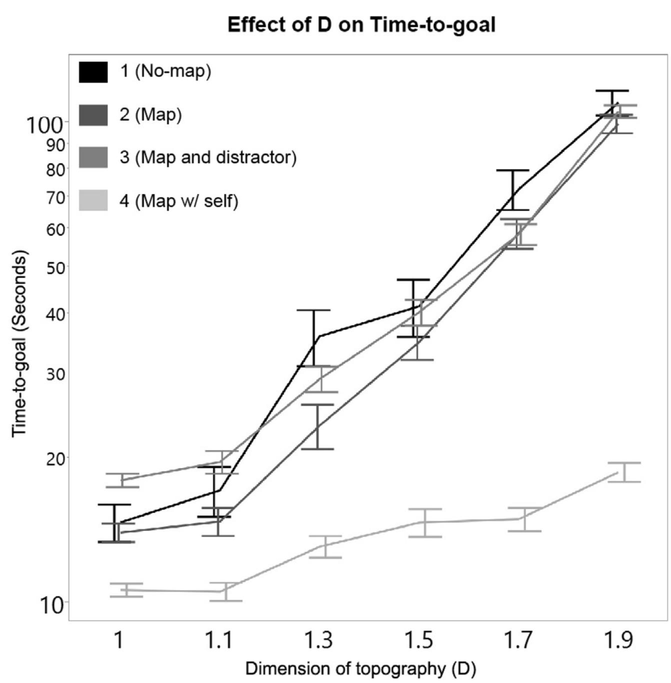

Fig. 5. Mean time to goal as a function of D for each of the four conditions plotted on a semi-log-scale. Error bars represent standard error of the mean. The error bars are slightly offset for visualization purposes.

Within Condition 3 (Map and distractor), the ratio of trials without errors (finding the goal without first arriving at the distractor) to total trials for each participant (i.e., the proportion of non-error trials) was analyzed as a function of D. A within subjects repeated measures ANOVA comparing the effect of D on the proportion of non-error trials in the six D conditions was conducted. Mauchly's test indicated that the assumption of sphericity was not violated; $X^2(14) = 18.75$, $p = 0.18$. The ANOVA revealed a significant effect of D; $F(5,135) = 4.98$, $p < 0.001$, $\eta^2 = 0.16$. Further Bonferroni corrected contrasts showed that there were significantly fewer non-error trials in the D = 1 condition than all others, $t = -3.48$, $p < 0.001$. We also found that D = 1.1 and D = 1.3 had significantly more non-error trials than D = 1.5–1.9 ($t = 2.76$, $p = 0.006$), suggesting that participants' best performances fell within the range $1 < D < 1.5$. These differences are presented in Fig. 6.

In order to account for both frequency of errors as well as time-to-goal within a single construct, we calculated a measure of overall performance as follows: standardized time-to-goal and standardized proportion of non-error trials were calculated, then summed and multiplied by −1. The result was a standardized measure of performance that took both measures into account, and that designated higher values as representing better performance. To determine the effect of landscape dimension on performance, we performed a within subjects repeated measures ANOVA with 6 levels of D. Mauchly's test indicated that there was no violation of the assumption of sphericity; $X^2(14) = 22.18$, $p = 0.06$. The results show that there was a significant effect of dimension on performance; $F(5, 135) = 25.64$, $p < 0.001$, $\eta^2 = 0.49$. Within subject contrasts showed significant linear ($p < 0.001$, $\eta^2 = 0.71$) and quadratic ($p < 0.001$, $\eta^2 = 0.56$) trends. Other higher-order trends were non-significant ($p > 0.05$ and $\eta^2 < 0.10$ for all other trends). Contrasts between D values indicated significantly greater performance for D = 1.1 and D = 1.3 when compared to all other dimensions; $t = 9.48$, $p < 0.001$. However, D = 1.1 and D = 1.3 were not significantly different from one another; $t = 0.42$, $p = 0.66$. The relationship between overall performance and D is presented in Fig. 7, revealing that performance increases as D decreases, with the highest performance at low-to-mid D values.

3.4. Discussion

The baseline condition (C4), which displayed the positions of both the participant and goal on the map, was intended to reveal the extent to which the experimental conditions, which only displayed the goal position (C2 & C3) or did not contain a map at all (C1), required knowledge of the environment and self-location relationship to effectively complete the task. As expected, in the

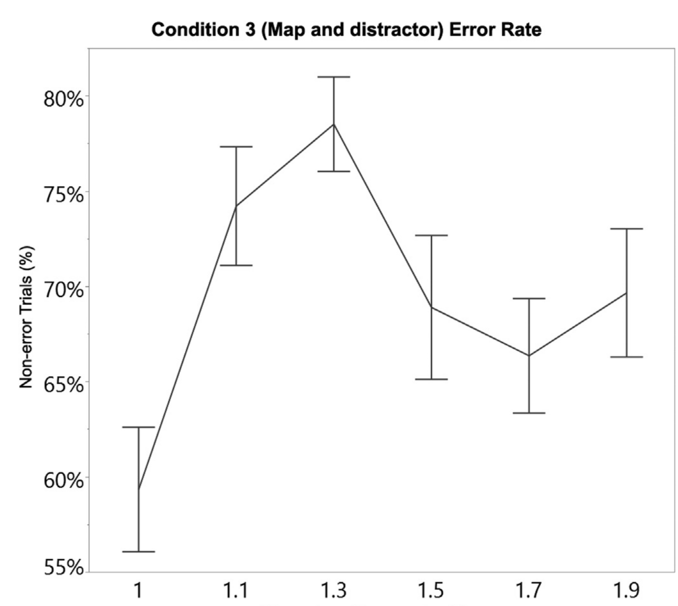

Fig. 6. Proportion of non-error trials as a function of D. Error bars represent standard error of the mean.

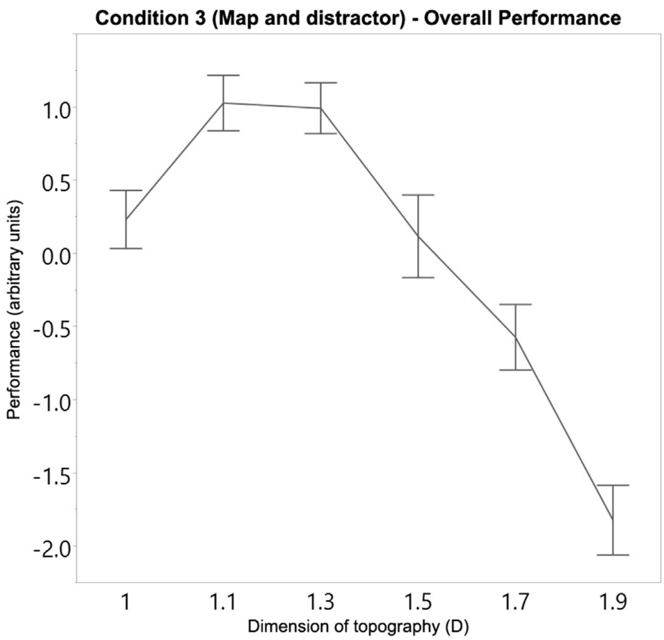

Fig. 7. Performance as a function of D for Condition 3 (Map and distractor). Performance takes into account both accuracy (frequency of errors) and reaction time (time-to-goal) and is defined as −1*(standardized time-to-goal + standardized proportion of non-error trials) where larger scores reflect higher performance. Error bars represent standard error.

absence of a need to explore the environment to discover the relative position of self and goal, participants completed trials in the baseline condition significantly faster than the experimental conditions. This confirmed that the experimental conditions indeed measured navigational performance above and beyond the difficulties of simply moving around the features of the environments. Additionally, our findings that the map conditions (C2 and C3) showed faster time-to-goal response times than the no-map condition (C1) suggests that participants were able to successfully utilize information within the map in order to complete a given trial.

The presence of exponential increases in time-to-goal within all three experimental conditions establishes consistency in our finding that navigation performance varies as a function of an environment's fractal dimension (D). The exponential increase in time to goal at the high D suggests that D indeed captures environmental complexity. Consistent with Stankiewicz et al. (2001), we find that performance decreases with high levels of environmental complexity. The contrasts conducted using our general performance measure point to peak performance in the mid-to-low range of D. However, the peak ($D = 1.1–1.3$) may be lower than the peak range for preference ($D = 1.3–1.5$) suggested in aesthetic experiments (Taylor et al., 2005, 2011) but fits with estimates of the ease of nameable shape perception (with greatest ease for $D = 1.2$) in object detection (Rogowitz & Voss, 1990). In our experiments, performance was highest when $1.1 \leq D \leq 1.3$.

One potential reason that we find terrains of $1.1 \leq D \leq 1.3$ as optimal for performance may be that the ability of participants to quickly complete a trial depended, in part, on how quickly the goal came into the line-of-sight of a participant for a given trial. Given the strong linear relationship between edge length and D, landscapes with lower D were statistically more likely to have large flat areas where the goal would be in the line-of-sight of the participant. With greater edge length, the extent to which any given view might be obscured by a feature of the landscape increases. In such cases where the goal is immediately visible to the participant, the benefit of information obtained from a map or any navigational strategy is unnecessary. This is apparent in the time-to-goal measure, with participants completing trials the fastest in the completely flat $D = 1.0$, and simple $D = 1.1$ topographies.

We attempted to correct for this aspect of the task by collecting a measure of error in Condition 3 (Map and distractor). Participants mistaking the distractor for the goal indicated an incomplete or incorrect understanding of the topography, and the error measure is able to capture this. Indeed, trials with the lowest landscape feature edge length ($D = 1.0$) showed the greatest number of errors. Additionally, the trend in proportion of non-error trials seen in Fig. 6 is consistent with the aesthetic and object-naming relationships described by others (Rogowitz & Voss, 1990; Taylor et al., 2005). As such, the combined performance measure attempts to partially correct for potential biases in time-to-goal responses that result from the influence of line-of-sight goal visibility. In Experiment 2, we directly address this limitation by employing an experimental design in which the goal is hidden and cannot appear within the line of sight.

4. Experiment 2: location judgment task

4.1. Purpose

The results of Experiment 1 suggest that the complexity of a terrain as described by its D value influences navigation ability. Given this finding, we designed Experiment 2 as both a replication and extension of the above work. First, we wanted to replicate the general pattern of results seen in Experiment 1 using a different set of fractal topographies and tasks. Additionally, we designed a task which would measure precision of spatial representation by eliminating the use of line-of-sight judgments. In the following experiment, participants made spatial judgments about the position of a hidden goal within the environment using a map and any spatial cues available to them within the virtual environment. The accuracy of these judgments was then related to the D of the landscapes in which they were made.

4.2. Method

4.2.1. Participants

Twenty-two (11 Males) new participants were recruited from the University of Oregon undergraduate research pool to participate in this experiment. No recruited individuals had participated in previous similar studies, and demographic information concerning major was not collected. This study was carried out in accordance with a protocol approved by the Research Compliance Services of the University of Oregon. All provided informed consent before undertaking the experiment. Ages of participants ranged from 19 to 38 years ($Mdn = 21$).

4.2.2. Procedure

The stimuli for Experiment 2 consisted of a different set of 30 island landscapes generated using the same parameters described in Section 2.1. In this experiment, participants were instructed to use a displayed map (similar to the ones shown in Fig. 3) to move their avatar to a goal position on the island as indicated by a red marker placed on the map. Crucially, an indication of this goal position was present only on the map, and not within the virtual environment itself. The goal position was verbally described to participants by an experimenter as the location of buried treasure. Using the controller configuration described in Section 3.2.2, participants indicated where they believed the goal position was located in the virtual environment by pressing a response button when they felt their avatar was standing at the position which corresponded to the marker on the map. They were then provided feedback on the accuracy of their judgment in order to encourage participants to not simply make random judgments but to be as accurate as reasonably possible for the duration of the experiment. Feedback consisted of a set of displayed text statements in various colors as well as a numerical point reward, both of which varied as a function of accuracy. The nature of each message is described in Table 2.

Once the judgment was made and feedback was given, a treasure chest then appeared in the environment at the true position of the goal, and participants were instructed to move toward it. During this post-judgment portion of the trial the map also indicated the participant's position, allowing for a timelier completion of each trial. Making contact with the goal object began a new trial on a new island. The order of islands as well as goal positions were randomized for each participant. Participants were instructed to complete as many trials as accurately as possible within the 50 min of the experiment. All participants completed at least 30 trials ($M = 65$, $Mdn = 71$). A short clip of this experimental design is presented in Video 2.

Supplementary video related to this article can be found at http://dx.doi.org/10.1016/j.jenvp.2016.05.011.

4.3. Results

Our measure of accuracy was designed to convey the ability of participants to make precise localization judgments on a scale that ranged from 0, designating chance performance, to 100, perfect performance. In order to accomplish this, we calculated judgment accuracy as '100 − distance in virtual meters between true and judged goal position.' Given that the diameter of each island is 200 virtual meters, an average error of 100 would equate to chance performance. By subtracting the judgment error from 100, we are able to achieve the desired scale. To determine the effect of landscape D value on judgment accuracy, we performed a within subjects repeated measures ANOVA with 6 levels of D. Mauchly's test indicated that the assumption of sphericity was not violated by the data distribution; $X^2(14) = 8.42$, $p = 0.87$. The results show that there was a significant effect of D on performance, $F(5, 140) = 9.19$, $p < 0.001$, $\eta^2 = 0.27$. Within subjects contrasts showed significant linear ($p = 0.03$, $\eta^2 = 0.16$) and quadratic ($p < 0.001$, $\eta^2 = 0.59$) trends. Other higher-order trends were non-significant ($p > 0.05$ and $\eta^2 < 0.10$ for all other trends). Further contrasts between individual D levels found that performance at D = 1.1, 1.3, and 1.5 was significantly greater than performance at all other levels ($t = 11.71$, $p < 0.001$). Performance scores at D = 1.3 were not significantly different from D = 1.1 ($t = 0.55$, $p = 0.58$), however they were significantly greater than D = 1.5 to a marginal extent ($t = 2.09$, $p = 0.045$). Peak judgment accuracy was within the low-to-mid D range, as can be seen in Fig. 8.

4.4. Discussion

We find the results of Experiment 2 to be consistent with those of the overall performance measure for Condition 3 (Map and Distractor) of Experiment 1, and both consistent with the original hypothesis that spatial judgments would be most accurate within the low-to-mid D range (D = 1.1–1.5). This result complements previous fractal perception findings for aesthetic judgments (Taylor et al., 2005, 2011) and object detection (Rogowitz & Voss, 1990). These similar performance relationships on separate tasks suggest that the results are likely not due to a particular set of fractal topographies or participants used in each experiment.

The absence of a linear relationship between D and accuracy suggests the importance and usefulness of the presence of a certain amount of complexity in a given environment for navigation tasks. We find near-chance performance both at the very low (D = 1.0) and high (D = 1.9) ends of D values, suggesting that these are points at which there is either too little or too much complexity to make meaningful judgments. These results can be interpreted in relation to concepts drawn from information theory, with the complexity of the environment conveying differing amounts of information that varies in a way similar to that of a classical entropy function (Reza, 1961). In this context, the low- and high-end D landscapes contain little information, whereas the low-to-mid D landscapes maximize information available to the participant.

In very low and very high D environments, the landscapes typically lack landmarks that meet the criteria established by Vinson (1999) to be meaningfully employed by a given individual. For example, both D = 1.0 and often D = 1.1 landscapes violated Vinson's (1999) guideline suggesting that an environment should contain more than one landmark. The guideline suggesting the need for distinctive landmark features is also violated at the high-end of D (1.9), where all features seem similar (see Fig. 3, bottom

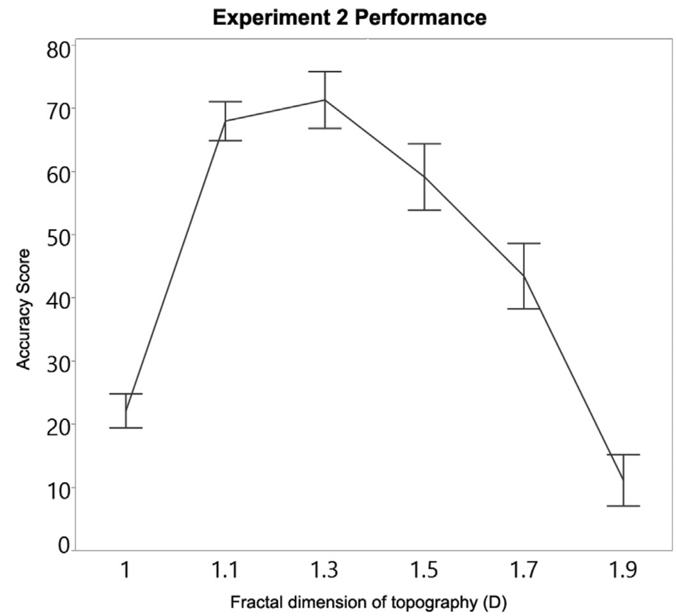

Fig. 8. The relationship between D and the mean accuracy. Accuracy was defined as 100 minus the difference in virtual meters between participant-judged and true goal positions. Error bars represent standard error.

Table 2
Descriptions of feedback provided to individuals after making a spatial judgment of goal position.

Distance from goal (in meters)	Verbal feedback	Reward amount	Text Color
0–10	"You found it!"	10 points	Green
11–30	"You were close."	5 points	Yellow
31–50	"You were not so close."	2 points	Red
50+	"You were way off."	0 points	Black

right panel). We find that the environments in which performance is optimal (landscapes with low-to-mid D) display features which consistently meet other criteria established by Vinson (1999) as well. The mixture of coarse and fine structure within most $D = 1.3$ landscapes, for example, results in landmarks which can be easily distinguished from each other, and also configurations in which more major landmarks are often flanked by one or more smaller landmarks.

The D value describes specific combinations of coarse and fine structure within an image, and as such, the results here support a range of ratios of structure (the ones found in low-to-mid D) as being important for goal-oriented navigation. An explanation for why these ratios might be ideal is that they may allow an individual to optimally utilize fine-to-coarse planning heuristics while moving through the environment (Wiener & Mallot, 2003). Both very high and very low D landscapes lack either the coarse structure needed for a coarse navigational strategy or the fine structure needed for a fine navigational strategy.

The greater frequency with which landscape features in the low-to-mid D range act as landmarks for participants also complements the findings of Bies et al. (2015) who show robust activation in brain regions responsible for object perception and recognition during the observation of low-to-mid D fractal images. An analogous pattern of activation may exist within brain regions associated with spatial perception and navigation, with specific coarse-to-fine arrangements of spatial information resulting in preferential activation of regions of the visual system responsible for landmark recognition. Janzen and Van Turennout (2004) found that the functionally-defined parahippocampal place area (PPA) shows preferential activation when viewing landmarks important to an active navigation task. Others have found that human subjects are sensitive to aspects of scene geometry (e.g., global structure and affordance properties; Greene & Oliva, 2009) which may be processed in scene-sensitive regions of cortex such as the PPA (Epstein & Kanwisher, 1998; Epstein, Harris, Stanley, & Kanwisher, 1999; Kravitz, Peng, & Baker, 2011; Park, Brady, Greene, & Oliva, 2011). This area and others may turn out to be important for navigation in low-to-mid complexity fractal environments.

5. Conclusion

Here we demonstrated the usefulness of D as a means of manipulating environmental complexity in navigational research. Furthermore, we have shown that in the context of accuracy measures of spatial judgment, results consistently support a specific range of D (1.1–1.3) as being optimal for goal-directed navigation. These results complement the results of other studies that have explored the effects of D (while viewing images that occupy 2-dimensional space) on aesthetic responses (Taylor et al., 2005, 2011), object detection (Bies et al., 2016; Rogowitz & Voss, 1990), discrimination and sensitivity (Spehar et al., 2015), and attention (Hagerhall et al., 2006). The current research extends the study of D to 3-dimensional settings and spatial perception. In doing so, the results support a theory of visual processing fluency in which stimuli of low-to-mid range D are easier to process for a variety of purposes.

We hope to extend the study of the relationship between an environment's D value and human navigation ability to other kinds of environments and navigational scenarios, such as driving within urban settings. Furthermore, variations in requirements of a given navigational task, such as line-of-sight and map reading demands, must be taken into account to understand the domains in which using D as a means of generating and measuring complexity is truly useful. As was observed in the difference in results between Experiments 1 and 2, there are potentially multiple relationships between D and performance measures that can appear given different task requirements. Additionally, while the experiments here were presented as traditional screen-based virtual environments, their generalizability to physical spaces must be explored as well.

By conducting such experiments in the future, we can better understand the ways in which humans are able to perceive and learn environments of varying complexity. The precise relationship between D and the human perceptual system's capacity to process spatial information could be utilized in a number of ergonomics and human factors contexts beyond natural landscapes, such as the design of constructed spaces, architecture or urban planning. This advantage of generalization can be seen in work extending fractal analysis beyond nature to constructed space, as has been done by Encarnação et al. (2012) when looking at urban topographies, with Stamps (2002) exploring aesthetic responses to fractal skylines, and with Taylor (2006) exploring physiological response to fractal architecture. Indeed, there already exists a rich tradition within architecture of incorporating fractal geometry into the layout of designed spaces. This history can be traced from Greco-Roman antiquity to Post-modern architecture (Goldberger, 1996; Salingaros, 1999), suggesting a natural inclination on the part of architects to design buildings using fractal geometry.

This natural inclination has been scientifically defended by multiple researchers who propose that the extent to which an urban space is fractal directly impacts the ability for that space to be coherent and livable (Joye, 2007; Salingaros & West, 1999). The work presented here extends this to the complexity within fractal layouts themselves, suggesting that certain levels of D allow for greater comprehensibility in navigation. Additionally, we provide a standard for future research measuring the D of fractal environments in relation to human performance. Having this simple, mathematically calculable method for characterizing the complexity of spatial environments may have long-term implications for environmental psychology and design.

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References

Aks, D. J., & Sprott, J. C. (1996). Quantifying aesthetic preference for chaotic patterns. *Empirical Studies of the Arts, 14*(1), 1–16.
Arthur, E. J., & Hancock, P. A. (2001). Navigation training in virtual environments. *International Journal of Cognitive Ergonomics, 5*(4), 387–400.
Balboa, R. M., & Grzywacz, N. M. (2003). Power spectra and distribution of contrasts of natural images from different habitats. *Vision Research, 43*(24), 2527–2537.
Bies, A. J., Kikumoto, A., Boydston, C., Greenfield, A., Chauvin, K., Taylor, R., et al. (2016). Percepts from noise patterns: the role of fractal dimension in object pareidolia. In *2016 vision sciences society meeting planner*. St. Pete Beach, FL: Vision Sciences Society, 2016. Online.
Bies, A. J., Wekselblatt, J., Boydston, C., Taylor, R. P., & Sereno, M. E. (2015). The effects of visual scene complexity on human visual cortex. In *2015 neuroscience meeting planner*. Chicago, IL: Society for Neuroscience, 2015. Online.
Bolliger, J., Sprott, J. C., & Mladenoff, D. J. (2003). Self-organization and complexity in historical landscape patterns. *Oikos, 100*(3), 541–553.
Chai, X. J., & Jacobs, L. F. (2009). Sex differences in directional cue use in a virtual landscape. *Behavioral Neuroscience, 123*(2), 276–283.
Chrastil, E. R., & Warren, W. H. (2013). Active and passive spatial learning in human navigation: acquisition of survey knowledge. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 39*(5), 1520–1537.
Darken, R. P., & Banker, W. P. (1998). Navigating in natural environments: a virtual environment training transfer study. In *Virtual reality annual international symposium, 1998. Proceedings, IEEE 1998* (pp. 12–19). IEEE.
Encarnação, S., Gaudiano, M., Santos, F. C., Tenedório, J. A., & Pacheco, J. M. (2012).

Fractal cartography of urban areas. *Scientific Reports, 2*, 527.
Epstein, R. A., Harris, A., Stanley, D., & Kanwisher, N. (1999). The parahippocampal place area: recognition, navigation, or encoding? *Neuron, 23*, 115–125.
Epstein, R. A., & Kanwisher, N. (1998). A cortical representation of the local visual environment. *Nature, 392*, 598–601.
Fairbanks, M. S., & Taylor, R. P. (2011). *Scaling analysis of spatial and temporal patterns: From the human eye to the foraging albatross. Non-linear dynamical analysis for the behavioral sciences using real data* (pp. 341–366). Boca Raton: CRC Press, Taylor and Francis Group.
Goldberger, A. L. (1996). Fractals and the birth of Gothic: reflections on the biologic basis of creativity. *Molecular Psychiatry, 1*(2), 99–104.
Gouyet, J. F., & Mandelbrot, B. (1996). *Physics and fractal structures*. Paris: Masson.
Greene, M. R., & Oliva, A. (2009). Recognition of natural scenes from global properties: seeing the forest without representing the trees. *Cognitive Psychology, 58*, 137–176.
Hagerhall, C. M., Laike, T., Kuller, M., Marcheschi, E., Boydston, C., & Taylor, R. P. (2015). Human physiological benefits of viewing nature: EEG response to exact and statistical fractal patterns. *Nonlinear Dynamics, Psychology, and Life Sciences, 19*(1), 1–12.
Hagerhall, C. M., Laike, T., Taylor, R., Küller, M., Küller, R., & Martin, T. (2006). Fractal patterns and attention restoration-evaluations of real and artificial landscape silhouettes. *IAPS, 19*.
Hagerhall, C. M., Laike, T., Taylor, R. P., Küller, M., Küller, R., & Martin, T. P. (2008). Investigations of human EEG response to viewing fractal patterns. *Perception, 37*(10), 1488–1494.
Hagerhall, C. M., Purcell, T., & Taylor, R. (2004). Fractal dimension of landscape silhouette outlines as a predictor of landscape preference. *Journal of Environmental Psychology, 24*(2), 247–255.
Janzen, G., & Van Turennout, M. (2004). Selective neural representation of objects relevant for navigation. *Nature Neuroscience, 7*(6), 673–677.
Joye, Y. (2007). Fractal architecture could be good for you. *Nexus Network Journal, 9*(2), 311–320.
Koch, M., Denzler, J., & Redies, C. (2010). 1/f2 characteristics and isotropy in the fourier power spectra of visual art, cartoons, comics, mangas, and different categories of photographs. *PLoS One, 5*(8), e12268.
Kravitz, D. J., Peng, C. S., & Baker, C. I. (2011). Real-world scene representations in high-level visual cortex: it's the spaces more than the places. *The Journal of Neuroscience, 31*(20), 7322–7333.
Loomis, J. M., Blascovich, J. J., & Beall, A. C. (1999). Immersive virtual environment technology as a basic research tool in psychology. *Behavior Research Methods, Instruments, & Computers, 31*(4), 557–564.
Malinowski, J. C., & Gillespie, W. T. (2001). Individual differences in performance on a large-scale, real-world wayfinding task. *Journal of Environmental Psychology, 21*, 73–82.
Mandelbrot, B. B. (1967). How long is the coast of Britain. *Science, 156*(3775), 636–638.
Mandelbrot, B. B. (1982). In *The fractal geometry of nature* (Vol. 173). Macmillan.
Moffat, S. D., Hampson, E., & Hatzipantelis, M. (1998). Navigation in a "virtual" maze: sex differences and correlation with psychometric measures of spatial ability in humans. *Evolution and Human Behavior, 19*(2), 73–87.
Montello, D. R., Sullivan, C. N., & Pick, H. L. (1994). Recall memory for topographic maps and natural terrain: effects of experience and task performance. *Cartographica, 31*, 18–36.
Morris, R. (1984). Developments of a water-maze procedure for studying spatial learning in the rat. *Journal of Neuroscience Methods, 11*(1), 47–60.
Nash, E. B., Edwards, G. W., Thompson, J. A., & Barfield, W. (2000). A review of presence and performance in virtual environments. *International Journal of Human-computer Interaction, 12*(1), 1–41.
O'Neill, M. J. (1992). Effects of familiarity and plan complexity on wayfinding in simulated buildings. *Journal of Environmental Psychology, 12*(4), 319–327.
Palmer, M. W. (1992). The coexistence of species in fractal landscapes. *American Naturalist*, 375–397.
Park, S., Brady, T. F., Greene, M. R., & Oliva, A. (2011). Disentangling scene content from spatial boundary: complementary roles for the PPA and LOC in representing real-world scenes. *The Journal of Neuroscience, 31*(4), 1333–1340.
Reza, F. M. (1961). *An introduction to information theory*. Courier Corporation.
Richardson, L. F. (1961). The problem of contiguity: an appendix of statistics of deadly quarrels. *General Systems Yearbook, 6*(13), 139–187.
Richardson, A. E., Montello, D. R., & Hegarty, M. (1999). Spatial knowledge acquisition from maps and from navigation in real and virtual environments. *Memory & Cognition, 27*(4), 741–750.
Rogowitz, B. E., & Voss, R. F. (1990). Shape perception and low-dimension fractal boundary contours. In *Human vision and electronic imaging: Models, methods, and applications* (Vol. 1249, pp. 387–394).
Salingaros, N. A. (1999). Architecture, patterns, and mathematics. *Nexus Network Journal, 1*(1–2), 75–86.
Salingaros, N. A., & West, B. J. (1999). A universal rule for the distribution of sizes. *Environment and Planning B: Planning and Design, 26*(6), 909–923.
Spehar, B., Clifford, C. W., Newell, B. R., & Taylor, R. P. (2003). Universal aesthetic of fractals. *Computers & Graphics, 27*(5), 813–820.
Spehar, B., & Taylor, R. P. (2013, March). Fractals in art and nature: why do we like them?. In *Society of photo-optical instrumentation engineers (SPIE) conference series* (Vol. 8651, p. 18).
Spehar, B., Wong, S., van de Klundert, S., Lui, J., Clifford, C. W. G., & Taylor, R. P. (2015). Beauty and the beholder: the role of visual sensitivity in visual preference. *Frontiers in Human Neuroscience, 9*, 514.
Stamps, A. E. (2002). Fractals, skylines, nature and beauty. *Landscape and Urban Planning, 60*(3), 163–184.
Stankiewicz, B. J., Legge, G. E., & Schlicht, E. (2001). The effect of layout complexity on human and ideal navigation performance. *Journal of Vision, 1*(3), 189–189.
Steck, S. D., & Mallot, H. A. (2000). The role of global and local landmarks in virtual environment navigation. *Presence: Teleoperators and Virtual Environments, 9*(1), 69–83.
Stürzl, W., Grixa, I., Mair, E., Narendra, A., & Zeil, J. (2015). Three-dimensional models of natural environments and the mapping of navigational information. *Journal of Comparative Physiology A, 201*(6), 563–584.
Taylor, R. P. (2006). Reduction of physiological stress using fractal art and architecture. *Leonardo, 39*(3), 245–251.
Taylor, R. P., Micolich, A. P., & Jonas, D. (1999). Fractal analysis of Pollock's drip paintings. *Nature, 399*(6735), 422–422.
Taylor, R. P., Spehar, B., Van Donkelaar, P., & Hagerhall, C. M. (2011). Perceptual and physiological responses to Jackson Pollock's fractals. *Frontiers in Human Neuroscience, 5*, 60.
Taylor, R. P., Spehar, B., Wise, J. A., Clifford, C. W., Newell, B. R., Hagerhall, C. M., et al. (2005). Perceptual and physiological responses to the visual complexity of fractal patterns. *Nonlinear Dynamics, Psychology, and Life Sciences, 9*, 89–114.
Tkacz, S. (1998). Learning map interpretation: skill acquisition and underlying abilities. *Journal of Environmental Psychology, 18*, 237–249.
Vinson, N. G. (1999). Design guidelines for landmarks to support navigation in virtual environments. In *Proceedings of the SIGCHI conference on human factors in computing systems* (pp. 278–285). ACM.
Voss, R. F. (1988). *Fractals in nature: From characterization to simulation* (pp. 21–70). New York: Springer.
Wiener, J. M., & Mallot, H. A. (2003). 'Fine-to-coarse' route planning and navigation in regionalized environments. *Spatial Cognition and Computation, 3*(4), 331–358.
Witmer, B. G., Bailey, J. H., & Knerr, B. W. (1995). *Training dismounted soldiers in virtual environments: route learning and transfer*. Orlando, FL: US Army Research Institute for the Behavioral and Social Sciences (ARI-TR-1022).
Wolbers, T., & Büchel, C. (2005). Dissociable retrosplenial and hippocampal contributions to successful formation of survey representations. *The Journal of Neuroscience, 25*(13), 3333–3340.