Dimensional Interplay Analysis of 'Poured' Paintings

1. Introduction

In 1999, Professor Richard Taylor's research team published results in the scientific journal *Nature* describing the 'poured' paintings of the Abstract Expressionist Jackson Pollock (see Reference 1 of *Appendix 1*). Their research used a scientific pattern analysis technique based on fractals. Fractal patterns are complex, organic-looking shapes that are prevalent in nature's scenery (see *Appendices 2* and *3* for a brief description of fractals). Holding professorships in physics, psychology and art, Taylor has published over 260 articles on fractals in various research disciplines throughout his 25-year career (physics, mathematics, engineering, psychology, physiology, geography, architecture and art). In addition to his PhD in Physics, Taylor has a Doctor of Science focusing on fractals and an Art Theory research degree on Pollock. On publication, the international art and science communities enthusiastically endorsed his findings. The Nobel Foundation (Sweden) invited Taylor to lecture on his Pollock research, as have art museums such as the Pompidou Centre (Paris) and the Guggenheim Museum (Venice). The Tate Modern gallery (London) asked Taylor to author the captions appearing underneath its Pollock paintings and Pollock-Krasner Study Center (USA) awarded him a research residency. Television companies such as PBS, ABC and the BBC have filmed documentaries featuring his research.

Fractal pattern analysis is an objective and accurate technique for analyzing Pollock's patterns (a fractal analysis involves up to 34 million pattern computations per painting). Among the enthusiastic supporters of Pollock's fractals was the 'father of fractals' Benoit Mandelbrot (1924-2010), who declared "I have extraordinary experience of these structures," concluding "I do believe that Pollock's are fractal". Other scientists have confirmed fractal analysis to be a useful approach to understanding Pollock's poured paintings. A recent list of peer-reviewed articles reporting fractal analysis of his paintings is presented in *Appendix 1*. This list is growing rapidly as more research groups build on Taylor's findings. As Jim Coddington (Conservator at New York's MOMA) noted in his paper on Pollock's fractal analysis: "In the visual arts we are at the beginnings of such a field. And make no mistake, it is coming" (see Reference 12 in *Appendix 1*). To date, 50 Pollock paintings have been analyzed by various groups, all of which have been well-described using fractals [see, for example, References 1 to 4 in *Appendix 1*].

The main thrust of Taylor's research on Pollock has focused on using fractal analysis to explore the similarities between Pollock's fractals and those found in Nature, with the aim of studying the aesthetic impact of fractals. This research indicates that exposure to fractals such as Pollock's paintings can induce positive psychological and physiological responses in the observer, such as reduced stress. An intriguing offshoot of his investigations is the study of poured paintings of unknown origin.

Significantly, Pollock's fractal patterns are not an inevitable consequence of pouring paint – they are produced by a specific pouring technique adopted by the artist. Taylor's analysis of Pollock's painting process identifies a highly systematic fractal generation process. Consequently, in order to replicate the visual characteristics of a Pollock painting it is not sufficient to simply generate fractals. The fractal painting should exhibit a specific set of trademark characteristics. With the aim of identifying and quantifying this set of characteristics, Taylor developed two fractal analysis procedures (in 2003 and 2006) known as *Dimensional Interplay Analysis (DIA) and Lacunarity Balance Analysis (LBA)*. These techniques represented a significant advance in the scientific interpretation of art: they used the artist's medical history to establish a link between the artist's specific physiology and the scientific parameters that quantify the resulting patterns. The two analysis techniques probe different characteristics of Pollock's fractal generation process. Consequently, an optimal understanding of a painting is obtained by integrating the results from both analysis techniques.

This document provides background information on the *DIA* technique. Of the 50 paintings that have undergone the general fractal analysis, Taylor's research team has performed the more detailed *DIA* on a subset of 14 paintings chosen to represent the variety of Pollock's poured catalogue (see *Section 2*). Based on these results, the two major Pollock authenticity organizations – the International Foundation for Art Research (IFAR) and the Pollock-Krasner Foundation - have used Taylor's *DIA* in conjunction with other important information such as provenance (i.e. historical documentation), connoisseurship (visual inspection by a Pollock scholar) and materials (paint, canvas and frame) analysis to investigate poured paintings of unknown origin. As summarized by chief Pollock scholar, Francis O'Connor, on his blog site (2011): "Taylor has demonstrated the analysis of the fractal structures in poured paintings by Pollock, that are consistent throughout his oeuvre, can be used to recognize poured paintings that are not from his hand."

2. DIA of Pollock's Fractal Patterns

Identification of Pollock's patterns is based on applying *DIA* to the following poured paintings (catalog numbers refer to those used in the *Catalogue Raisonné* by F.V. O'Conner and E.V. Thaw): *Untitled* (1945) (no. 990), *Full Fathom Five* (1947) (no. 180), *Lucifer* (1947) (no. 185), *Number 23, 1948* (1948) (no. 199), *Number 14, 1948* (1948) (no. 204), *Figure* (1948) (no. 783), *Number 8, 1949* (1949) (no. 239), *Number 27, 1950* (1950) (no. 271), *Number 32, 1950* (1950) (no. 274), *Autumn Rhythm: Number 30* (1950) (no. 297), *Unknown on Glass* (1950), *Untitled* (1951) (no. 825), *Convergence: Number 10, 1952* (1952) (no. 363), and *Blue Poles: Number 11, 1952* (1952) (no. 367). Although these 14 works represent a subset of Pollock's catalogue, the paintings were carefully selected to represent the diversity of his poured paintings (this selection was approved by the Pollock Krasner Foundation). This sub-group ranges from one of his smallest paintings (e.g. 57.5 by 78.4cm of *Number 23, 1948*) to one of his largest canvases (*e.g.* 266.7 by 525.8cm of *Autumn Rhythm*) and includes paintings created using different paint media (enamel, aluminum, oil, ink and gouache) and supports (canvas, cardboard, paper and glass).

DIA is based on the hypothesis that Pollock's poured abstract paintings are typically composed of two sets of fractal patterns. *DIA* indicates that these so-called 'bi-fractal' patterns exhibit the following typical characteristics:

1. The two sets of fractal patterns dominate over different size scales. The patterns observed at 'large' size scales were generated predominately by Pollock's body motions. The patterns observed at 'small' size scales were generated predominately by the manner in which the paint fell through the air and impacted the canvas surface (i.e. the 'pouring' process).

2. The size scale marking the transition between the two sets of patterns is labeled L_T. Although the value of L_T varies between Pollock's paintings (depending on factors such as the canvas size), L_T is typically not less than 1cm. The average of all the measured values of L_T from the 14 paintings is 3.0cm. This transition is consistent with the physical origins of the two fractal processes (i.e. Pollock's motions and the pouring process).

3. The two sets of fractal patterns are each well-described by a parameter called the *Effective Fractal Dimension*. This parameter is labeled as *D* and is described in more detail in *Appendices 2* and *3*. Fractal dimension lies at the center of fractal research - it contains information about how the patterns observed at different magnifications combine to build the observed painting. Significantly, the *D* value has a profound impact on the visual character of

a pattern. In particular, the higher the D value, the higher the pattern's visual complexity. For Pollock's paintings, the D value typically lies on a scale between 1 and 2: patterns closer to 1 are simple and sparse (low complexity) and those closer to 2 are rich and intricate (high complexity).

4. Because the two sets of fractal patterns are generated by different physical processes (his physical motions and his pouring process), they are typically characterized by different D values: the D value describing the pouring process (labeled D_D) is lower than the D value of the fractal patterns generated by his motions (labeled D_L). This 'bi-fractal' interplay of visual complexity across two distinct size scales makes the paintings distinguishable.

5. Unlike their mathematical counterparts, bi-fractals in the physical world (such as those produced by Pollock) reveal measurable variations from 'idealized' scaling behavior. For example, the physical pattern evolves gradually between the two fractals rather than switching over instantly at L_T. Other physical distortions at various magnifications might also generate variations. For Pollock paintings, the computer quantifies these variations using a parameter called the variation. The variation is labeled sd and is described in more detail in *Appendix 3*. Patterns that more closely follow idealized fractal behavior are characterized by low sd values (close to 0) and the value increases for patterns exhibiting larger variations. For the analyzed Pollock paintings, the sd value lies in the range 0.009 to 0.027. Ongoing research of Pollock paintings suggests that the sd value reduces with canvas size.

6. For paintings containing more than one poured color, the patterns formed by the different colors can be separated and analyzed individually. If there are technical limitations when separating a colored layer, the layer is not analyzed. *DIA* of the 14 analyzed paintings suggests that the above characteristics typically apply to all layers.[+] Note that layers within the same painting may differ in their D_D, D_L and L_T values.

3. Poured paintings created by artists other than Pollock

Poured paintings produced by 37 undergraduates have been analyzed. NONE of these paintings matched the 6 characteristics presented in *Section 2*.

4. Analysis of poured paintings of unknown origin

For a variety of reasons, it may be informative to compare the fractal characteristics of poured paintings of unknown origin with poured paintings attributed to Pollock. Due to the large number of requests for *DIA* analysis, Taylor formed a research company called *Fractals Research LLC* to conduct research of poured paintings. Based on the current status of Taylor's research, the following *DIA* procedure is adopted when making this comparison.

As with Pollock's paintings, the patterns formed by different colors are separated and analyzed individually. For each colored pattern, the computer 'fits' the pattern to the 'bi-fractal' behavior outlined in *Section 2* and generates the four parameters L_T, sd, D_D and D_L. A 'free fit' procedure is first applied to the layer being analyzed, in which the computer searches for the L_T value that gives the lowest sd value for the layer. If the resulting L_T value lies within the range observed for

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[+]Note: The *DIA* criteria are only reliable when employing the hybrid color-separation technique developed by Taylor's research group. Furthermore, the criteria are intended only for abstract poured paintings. Also, note that the background (usually established by the exposed canvas) pattern is not required to be fractal.

a typical Pollock pattern (equal to or above 1cm) then this is termed a 'free fit'. As a general guide, the magnitude of sd should be less than 0.030 to be considered a match to typical Pollock patterning. The computer then calculates the two D values, which should lie between 1 and 2 and the D_L value should also to be larger than the D_D value (as typically observed for Pollock's patterns).

If the 'free fit' procedure fails to produce a L_T value that equals or lies above 1cm, the layer is given 'a second chance' by using a 'forced fit'. Adopting this procedure, the computer fits the patterns to the 'bi-fractal' behavior using the L_T value of 3.0cm (i.e. the average L_T value of the Pollock patterns analyzed) and then calculates the sd value. Using this procedure, as a general guide the value of sd should be less than 0.030 to be considered a match to Pollock patterning. The D values should lie between 1 and 2 and D_L is also required to be larger than the D_D value (i.e. the criteria for sd, D_D and D_L are the same as for the 'free fit').

Note that, for both the 'free' and 'forced' fits, the 'match' range for sd (i.e. lying between 0 and 0.029) is wider than the range exhibited by the established Pollocks (for which sd is between 0.009 and 0.027). Paintings with sd values lying between 0.027 and 0.029 are considered to be sufficiently close to the patterns found in established Pollocks to warrant interest and are therefore considered a 'match' in terms of the sd value. Note also that, according to on-going research, smaller paintings might be expected to have lower sd values than larger paintings.

5. Summary of 'match' behavior to typical Pollock characteristics

For a layer to be considered a 'match' to the typical fractal characteristics of a Pollock painting, the D_D and D_L values should be between 1 and 2 (with the D_L value higher than the D_D value) and the sd value should be less than 0.030. The L_T value should be equal to or greater than 1cm.

6. Common behavior for paintings of unknown origin

Colored layers extracted from paintings of unknown origin exhibit a variety of interesting fractal characteristics. These include:

1. The sd value is large ("$sd > 0.029$")
2. The D_D or D_L value is low ("D_D or D_L not in the range $1.05 \leq D < 2$")
3. The D_D and D_L values are close, such that the pattern is not considered to be bi-fractal ("$|D_D - D_L| < 0.05$")
4. The D_D value is larger than the D_L value, such that the pattern is an inverted bi-fractal ("$D_D > D_L$")
5. The D_L value is observed over a limited magnification range (no. pts. ≤ 5)

7. Interpretation of *DIA* Results

For each painting, *Fractals Research* issues a report containing the following information: 1) the scaling plots (described in *Appendix 3*) showing the results of the *DIA* analysis for each colored layer that could be reliably separated, 2) a table summarizing the parameters described in *Sections 2* and *4* for each layer, and 3) a brief description of any interesting facts. For example, these facts might include characteristics listed in *Section 6* (in which case, the statements appearing in parentheses will be reported). The facts might also include reasons why specific layers could not be color-separated.

It is important to note that the reported results can be used for a scientific comparison of the fractal characteristics of poured paintings. Taken in isolation, these results are not intended to be a technique for attributing a poured painting to Pollock. Note, however, that the results may be useful in this process when coupled with other important information such as provenance (i.e. historical documentation), connoisseurship (visual inspection by a Pollock scholar) and materials (paint, canvas and frame) analysis.

Prior to 2006, reports also summarized the findings by placing the painting in one of the following categories, based on the 6 characteristics outlined in *Sections 2* and *4* (and summarized in *Section 5*):

(a) "The fractal characteristics of the painting closely match those of Pollock's paintings." For these paintings, each of the colored patterns matches all of the outlined typical characteristics.

(b) "The characteristics of the painting show deviations from the fractal characteristics of Pollock's paintings." At least one of the outlined typical characteristics has not been matched for at least one of the colored layers.

This 2-group categorization served only as a guide and is no longer used because it unintentionally encouraged an over-simplification of the results. In particular, *Fractals Research* strongly discourages a "green-light/red-light" interpretation of *DIA* results. A match to the criteria (previously assigned by *Fractals Research* as Category A) does not *automatically* mean that the painting is by Pollock. Similarly, Category B does not *automatically* mean that a painting is not by Pollock. Furthermore, *Fractals Research* emphasizes the wealth of information in the table of analyzed parameters presented in the reports - the 'categorization/match' approach based on the 6 outlined characteristics is just one approach to using this information to study the analyzed poured painting.

Nevertheless, the hypothesis that the 6 outlined characteristics are typical of Pollock paintings is supported by initial empirical evidence. For example:

1. In 2003, when IFAR used *DIA* to investigate poured paintings of unknown origin, the results matched the views of their visual experts.

2. In 2005, *DIA* produced a Category B result for a controversial poured work referred to as the "Parker" painting. It was subsequently revealed that an artist other than Pollock had been seen painting the work [see "the Blue Print", *ArtNews*, June 2008].

3. In 2006, the Pollock-Krasner Foundation applied *DIA* to 6 of the "Matter" painting collection [see "Computer Analysis Suggests Paintings are Not Pollocks", *New York Times*, February 9th, 2006], and the Category B result is consistent with subsequent materials analysis revealing the presence of paint pigments dating from after Pollock's death.

Despite these consistent results, *Fractals Research* notes that fractal analysis is in its infancy in regard to art interpretation and a number of different analysis techniques are currently being developed by *Fractals Research* and other research groups to supplement *DIA* (see *Appendix 1*). In particular, it is expected for *DIA* to be integrated with Lacunarity Balance Analysis (*LBA*), traditional visual inspection, materials analysis and provenance information.

8. Image Requirements

To perform a reliable analysis, high quality source images are required.[#] Clients are responsible for submitting electronic images that match, as closely as possible, the target image characteristics. These targets are as follows. Accurate values of the physical dimensions (height and length) of the painting are required (accurate to within 1mm). The painting should be photographed using uniform lighting conditions and with the plane of the painting oriented parallel to the observer (i.e. not at a slant). An optimal analysis requires features with a physical size of 1mm to be both in focus and photographed at a resolution of at least 5 pixels per mm on the canvas. Fractals Research would prefer the photograph to be of the whole painting. However, if the above conditions can only be achieved by photographing a sub-section of the painting then this is acceptable providing the section is as large as possible (0.75m by 0.75m or larger) and that the chosen subsection is similar in character to other subsections. In the case where a sub-section is submitted for analysis, Fractals Research will also require a low-resolution photograph of the whole painting.

Fractals Research restricts its analysis to the colored layers that can be successfully extracted from the painting. High quality source images help to maximize the number of colored layers that can be extracted. Fractals Research does not take responsibility for distortions in the analysis originating from image limitations.

Appendix 1: A summary of fractal analysis techniques applied to Pollock's patterns
The following is a list of peer-reviewed articles that have built of Taylor's findings and have demonstrated the useful application of fractal analysis to Pollock's poured paintings:

1. R.P. Taylor, R. Guzman, T.P. Martin, G.D.R. Hall, A.P. Micolich, D. Jonas, B.C. Scannell, M.S. Fairbanks, C.A. Marlow
"Authenticating Pollock Paintings Using Fractal Geometry"
Publication status: *Pattern Recognition Letters*, vol. 28, 695-702 (2005), *Nature*, vol. 399, 422 (1999), *Nature*, vol. 444, E10-11 (2006).
Technique: Dimensional interplay analysis (DIA) of color-separated layers.

2. R.P. Taylor,
"Order in Pollock's Chaos",
Publication Status: *Scientific American*, vol. 287, 116-121 (2002)
Technique: Derivative analysis of scaling behavior of color-separated layers

3. R.P. Taylor, A.P. Micolich and D. Jonas
"The Construction of Fractal Drip Paintings"
Publication Status: *Leonardo*, vol. 35, 203-207 (2002) and in "Art and Complexity" (Eds. Ed. J. Casti and A. Karlqvist, Elsevier Science, Amsterdam (2003).
Technique: Box-counting analysis applied to paintings photographed at various stages of the painting process.

4. M. Fairbanks, J.R. Mureika, R.P. Taylor et al
"The Art of Balance: Using Fractal Analysis to Investigate the Role of Physiology in the Artistic Generation of Poured Paintings"
Publication status: Manuscript in preparation
Technique: Lacunarity analysis

5. J.R. Mureika, M. Fairbanks, R.P. Taylor
"Multifractal Analysis of the Painting Techniques of Adults and Children"
Publication status: Published: *SPIE Proceedings of Electronic Imaging*, special edition on "Computer Vision and Image Analysis of Art", Ed. D.G. Stork, J. Coddington and A. Bentkowska-Kafel, *vol.* 7531 7531001-6 *(*DOI: 10.1117/12.840411) (2010*)*
Technique: Multi-fractal analysis of edges of grayscale images.

6. J.R. Mureika, C.C. Dyer, G.C. Cupchik,
"Multifractal Structure in Nonrepresentational Art"
Publication status: *Physical Review E*, vol. 72, 046101-1-15 (2005) and *Leonardo*, vol. 37, 53-56 (2004).
Technique: Multi-fractal analysis of edges of grayscale images.

7. J.R. Mureika
"Fractal Dimensions in Perceptual Color Space: A Comparison Study Using Jackson Pollock's Art"
Publication status: *Chaos*, vol. 15, 043702-1-6 (2005).
Technique: Fractal analysis of colored layers separated using perception color selection (in contrast to physical color separation employed in refs. 2 and 3).

8. D.J. Graham and D.J. Field

"Variations in Intensity for Representative and Abstract Art, and for Art from Eastern and Western Hemispheres"
Publication status: *Perception*, vol. 37, 1341-1352 (2008).
Technique: Fast Fourier transform (FFT) analysis of grayscale images
See, also, "Statistical Regularities of Art Images and Natural Scenes: Spectra. Sparseness and Non-Linearities", D.J. Graham and D.J. Field, *Journal of Spatial Vision*, vol. 21, 149-164 (2007).

9. C. Redies, J. Hasenstein and J. Denzler
"Fractal-Like Image Statistics in Visual Art: Similar to Natural Scenes"
Publication status: *Spatial Vision*, vol. 21, 137-148 (2007).
Technique: Fast Fourier transform (FFT) analysis of grayscale images.
See, also, C. Redies "A Universal Model of Esthetic Perception Based on the Sensory Coding of Natural Stimuli" *Spatial Vision*, vol. 21, 97-117 (2007).

10. J. Alvarez-Ramirez, C. Ibarra-Valdez, E. Rodriguez and L. Dagdug
"1/f-Noise Structure in Pollock's Drip Paintings"
Publication status: *Physica A*, vol. 387, 281-295 (2008).
See also J. Alvarez-Ramirez, E. Rodriguez, R. Escarela-Perez "On the Evolution of Pollock's Paintings Fractality"
Technique: Detrended fluctuation analysis (DFA) to grayscale images.

11. J. Alvarez-Ramirez, J. C. Echeverria, E. Rodriguez
 "Performance of a High-Dimensional R/S Analyis Method for Hurst Exponent Estimation"
Publication Status: *Physica A*, vol. 387, 6452-6462 (2008).
Technique: High-Dimensional R/S method for Hurst Estimation applied to grayscale image

12. S. Lee, S. Olsen and B. Gooch
"Simulating and Analyzing Jackson Pollock's Paintings"
Publication status: *Journal of Mathematics and the Arts*, vol.1, 73-83 (2007).
"Interactive 3D Fluid Jet Painting"
Publication: "Non-Photorealistic Animation and Rendering", *Proceedings of the 4th International Symposium on Non-Photorealistic Animation and Rendering*, 97-104, (2006) ISBN: 1-59593-357-3.
Technique: Interactive technique for simulating a poured painting based on Navier-Stokes equations and fractal analysis of the pattern.

13. J. Coddington, J. Elton, D. Rockmore and Y. Wang
"Multi-fractal Analysis and Authentication of Jackson Pollock Paintings"
Publication status: *Proceedings SPIE*, vol. 6810, 68100F 1-12 (doi:10.1117/12.765015) (2008).
Technique: Multi-fractal analysis of grayscale images, in particular investigation of entropy dimension.

14. M. Irfan and D. Stork
"Multiple Visual Features for the computer authentication of Jackson Pollock's drip paintings: Beyond Box counting and Fractals"
Publication: SPIE Electronic Imaging: Machine vision applications II, Eds. K.S. Niel and D.Fofi vol. 7251, 72510Q1–11 (2009).
See also M. Al-Ayyoub, M. T. Irfan and D.G. Stork, "Boosting Multi-Feature Visual Texture Classifiers for the Authentification of Jackson Pollock's Drip Paintings", SPIE proceedings on Computer Vision and Image Analysis of Art II, Ed.s D.G. Stork, J. Coddington and A.

Bentkowska-Kafel, vol 7869, 78690H, and see D. Stork "Comment on "Drip Paintings and Fractal Analysis: scale space in multi-feature classifiers for drip painting authentication"
Technique: Integration of multiple pattern analysis techniques, including box counting fractal analysis.

15. R.P. Taylor, B. Spehar, P. Van Donkelaar and C.M. Hagerhall
"Perceptual and Physiological Responses to Jackson Pollock's Fractals"
Publication status: frontiers in Nueroscience, 5 1- 13 (2011).
Technique: Measurement of physiological responses to Pollock artworks.

16. A. Herczynski, A. C. Cernuschi, and L. Mahadevan,
"Painting with Drops, Jets, and Sheets
Publication status, Physics Today, June edition, 31-36 (2011).

The following articles have discussed potential limitations of fractal analysis. Taylor's published replies are indicated below each article.

17. C. Cernuschi, A. Herczynski and D. Martin,
"Abstract Expressionism and Fractal Geometry",
Publication status: Non-refereed chapter of "Pollock Matters" by E. Landau and C. Cernuschi (Eds.), McMullen Museum of Art, Boston College, 91-104 (2007).
Technique: Derivative Analysis of color-separated layers (same technique as in Ref 2.)
COMMENT: the authors interpreted their fractal analysis in terms of so-called 'arc-fractals'. However, this interpretation is inconsistent with the painting process used by Pollock. In contrast, the bi-fractal picture presented by Taylor et al is physically reasonable.

18. K. Jones-Smith et al.
"Fractal Analysis: Revisiting Pollock's Paintings"
Publication status: *Nature, Brief Communication Arising*, vol. 444, E9-10, (2006).
See also *Physical Review E*, vol. 79, 046111 (2009).
Technique: Dimensional interplay analysis (DIA) of color-separated layers.
COMMENT: this investigation did not use the appropriate color separation procedure, nor did the authors take the necessary steps to maximize the standardization of their analysis (as outlined in Ref. 1). Consequently, their work represents a misapplication of the DIA technique: it is not therefore clear if their results highlight limitations of the DLA technique or just their specific application of the technique.

The arguments in [18] were addressed in Taylor's replies: *Nature*, vol. 444, E10-11 (2006) and arXiv:0712.1652v1 [*cond-mat.stat-mech*] 2007 (2009). Below are published comments on Jones-Smith et al's research by other fractals researchers:
"What Jones-Smith and Mathur have done is just a simple trick – this is bad science about fractals" (Chaos expert, Prof Lansaros Gallos, City College of New York, *ScienceNews*, February 2006)."
"The criticisms and negative recommendations [of Jones-Smith et al] arose from a failure to follow well-established principles and methodologies from statistical pattern recognition" (David Stork in Ref. 14 above). Stork stressed that their criticisms "were not relevant to the design of classifiers for Pollock authenticity." He concluded, "It is reasonable to extract some measure of fractal behavior as a feature of drip painting."
"The method of box-counting and more generally fractal geometry has begun to play an important role in the authentication of the work of Jackson Pollock. We believe such [fractal] analyses are necessary for pushing the field forward." Art conservator J. Coddington and computer scientist D. Rockmore (Ref. 13 above).

"I think they [Jones-Smith et al] took a fairly simplistic way of separating those colors which could have skewed their results" (Hany Farid, professor, Dartmouth College, Scientific American, http://www.scientificamerican.com/article.cfm?id=can-fractals-spot-genuine).
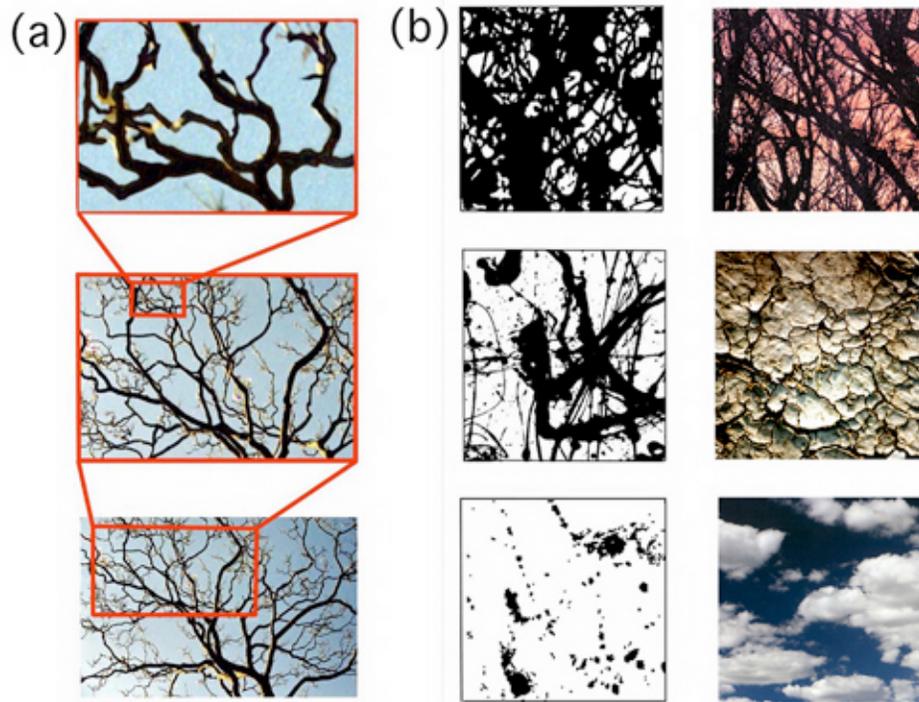
Appendix 2: an introduction to fractal patterns

Figure 1: (a) The repeating patterns of a fractal tree, shown at three magnifications, (b) Fractal patterns and their D values. Left column, from bottom to top: three fractal poured paintings with D=1.1, 1.7 and 1.9. Right column, bottom to top: clouds (D=1.3), mud cracks (D=1.7) and a forest of trees (D=1.9).

A spectacular variety of natural objects are fractal, earning them the popular title of the 'Fingerprint of Nature'. The shapes of fractal objects are formed by patterns that repeat at increasingly fine magnifications. More specifically, the statistical qualities of the pattern repeat, with the consequence that patterns viewed at different magnifications look similar to each other. This repetition creates immense visual complexity. Even the most common fractal objects, such as the tree shown in Figure 1(a), contrast sharply with the simplicity of artificial 'Euclidean' shapes such as squares, circles and triangles. Other common examples of nature's fractals are mountains, rivers, clouds and lightning.

An important parameter for quantifying a fractal pattern's visual complexity is the effective fractal dimension, D. This parameter describes how the patterns occurring at different magnifications combine to build the resulting fractal shape. For Euclidean shapes, dimension is described by familiar integer values - for a smooth line (containing no fractal structure) D has a value of one, while for a completely filled area (again containing no fractal structure) its value is two. For the repeating patterns of a fractal line, D lies between one and two and, as the complexity and richness of the repeating structure increases, its value moves closer to two. For fractals described by a low D value, the patterns observed at different magnifications repeat in a way that builds a very smooth, sparse shape. However, for fractals with a D value closer to two, the repeating patterns build a shape full of intricate, detailed structure. Figure 1(b) demonstrates how a pattern's D value has a profound effect on the visual appearance.

Appendix 3: an introduction to fractal analysis and scaling plots

Figure 2 shows a schematic representation of a fractal analysis of a poured painting. The analysis employs the 'box-counting' method, making it particularly robust to limitations arising from source image resolution and color-separation irregularities. The digitized image of the paint layer is covered with a computer-generated mesh of identical squares (or 'boxes') (see Reference 1 of *Appendix 1* for details). The statistical scaling qualities of the pattern are then determined by calculating the proportion of squares occupied by the painted pattern (shaded blue in Figure 2) and the proportion that are empty. This process is then repeated for increasingly fine meshes (i.e. smaller square sizes). Reducing the square size is equivalent to looking at the pattern at finer magnification. In this way, the pattern's statistical qualities can be compared at different magnifications. Specifically, the number of squares $N(L)$ that contain part of the painted pattern are counted and this is repeated as the size, L, of the squares in the mesh is reduced. For fractal behavior, $N(L)$ scales according to the power law relationship $N(L) \sim L^{-D}$, where D lies between 1 and 2. This power law generates the scale invariant properties that are central to fractal geometry.
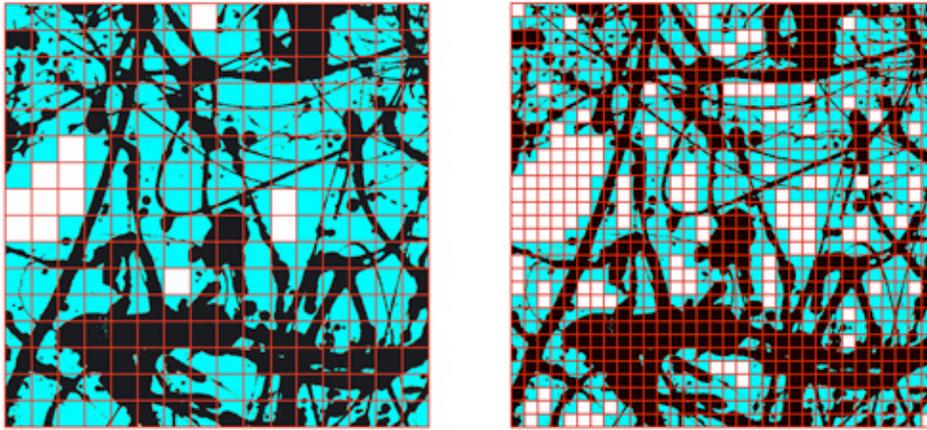

Figure 2: 'Box-counting' fractal analysis of a poured painting.

The graph shown in Figure 3(a) is for a paint layer extracted from *Blue Poles* (1952). This graph is representative of the typical fractal characteristics of the Pollock poured paintings that have been analyzed to date. Based on the current status of research, these characteristics are therefore adopted as an indication of the artistic 'trademark' of Pollock's poured patterns.

The graph is a plot of log N versus log L (where L is measured in millimeters) and is referred to as a scaling plot. The horizontal axis represents pattern size, which decreases from left to right. In other words, the data associated with the largest pattern is plotted on the extreme left, and that of the smallest pattern is plotted on the extreme right. The dots represent data points obtained from the box-counting analysis. For a fractal painting, these data points must lie on a sloping straight line. Because there are two sets of fractal patterns in Pollock paintings (one due to the paint pouring process and one due to his motions across the canvas) the data is expected to follow two lines and this can be seen to be the case – the scaling plot is therefore consistent with bi-fractal behavior. The computer has fit a straight line through the left hand data and another line through the right hand data. The transition size, L_T, between the two sets of fractals can be extracted from this graph and is measured as 1.8cm. The slope of the fitted line gives the value of D of the fractal pattern. For the right-hand slope (the pouring process at small size scales) the D_D value is 1.63. For the left-hand slope (the motion process at large size scales) the D_L value is 1.96.
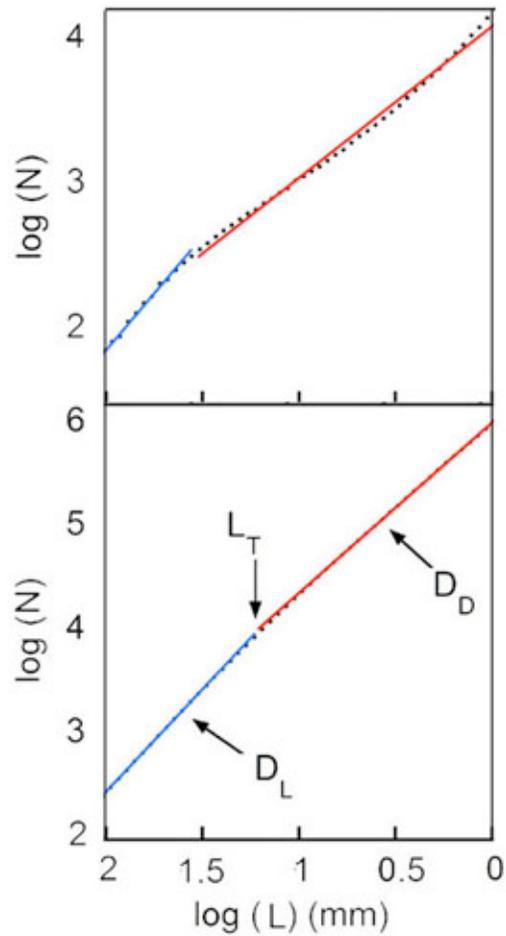
Figure 3. Bottom: a scaling plot for *Blue Poles*, displaying typical fractal characteristics of Pollock's fractal patterns. Top: a scaling plot of a poured painting by another artist.

The computer assesses how closely the data points follow the two lines using a parameter called the variation *sd*. The value of *sd* increases as the data points deviate from the two fit lines (i.e. the data points no longer lie directly on the lines). To standardize the *sd* value, *DIA* is applied across the log (*L*) range between 2 (corresponding to $L = 10$cm) and 0 (corresponding to $L = 1$mm). The computer gives the *sd* value for this graph as 0.020, which indicates a very low deviation (i.e. the data points lie very close to the fit lines). For comparison, we show Figure 3(b) as an example of a scaling plot for a poured painting by an artist other than Pollock. Here the data points no longer lie directly on the two lines. The deviation of the data points from the lines is measured as *sd* = 0.038.