The Sinai Light Show: Using Science to Tune Fractal Aesthetics

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Introduction

Nature's beauty is profound. To better understand the source of this beauty, here we will focus on the aesthetic impact of fractals. Fractals are patterns that repeat at increasingly fine size scales and they are prevalent throughout nature's scenery [1]. Examples include lightning, clouds, trees, rivers and mountains. Furthermore, they have permeated cultures spanning across many centuries and continents, ranging from Hellinic friezes (300 B.C.E) to Jackson Pollock's poured paintings (1950s) [2-4]. We will discuss how science can be used to determine the origin of fractal aesthetics and also to generate patterns that maximize this aesthetic experience.

Fractals play a central role in our visual experiences because the human visual system has adapted to these prevalent natural patterns. We will review our experiments showing that this adaption influences many stages of the visual system. Based on these results, we will present a 'fractal fluency' model in which the visual system processes the visual properties of fractals with relative ease. This fluency optimizes the observer's skill at performing visual tasks (for example, leading to enhanced pattern recognition capabilities) and generates an aesthetic experience accompanied by a reduction in the observer's physiological stress-levels.

Having established the visual mechanism underlying fractal aesthetics, we will then refine the fractal characteristics to amplify their visual impact. Although computer-generated fractals are well-known within the art world, we will adopt a more natural process to achieve our goal. Many of nature's fractal objects are experienced through light effects – for example, the shadows of clouds, mountains and trees pervade our daily experiences. Inspired by this prevalence of light patterns, we will reflect rays of light between multiple mirrors to build light patterns at many scales (Fig.1). Known as a Sinai billiard [5], the apparatus we will use consists of a cube of mirrors with a spherical mirror positioned at its center. Reflection off the curved surfaces induces chaos in the light rays and this leads to fractal light patterns.

By adjusting the mirrors, the fractal characteristics of the observed pattern can be evolved. In particular, the relative amounts of coarse and fine structure in the fractal can be changed. Significantly, the resulting evolution in the pattern's visual complexity is central to its fractal aesthetics. Consequently, we can use this system to increase the aesthetic quality of the pattern. Crucially, we will show that, although there are universal preferences shared by all observers, there are also factors that cause subtle differences between observers. Hence, our system is ideal for tuning the aesthetics to the needs of the individual observer.

The Visual Impact of Fractals

In Fig. 2, we use trees to demonstrate the intrinsic visual properties of fractals. Fractals fit into 2 categories – 'exact' (left image) and 'statistical' (right image). Whereas exact fractals are built by repeating a pattern at increasingly fine magnifications, 'statistical' fractals introduce randomness into their construction. This disrupts the precise repetition so that only the pattern's statistical qualities (e.g. density, roughness, and complexity) repeat. Consequently, statistical fractals simply look similar at different size scales. Whereas exact fractals exhibit the cleanliness of artificial shapes, statistical fractals reveal the organic signature of nature's scenery.

Statistical fractals feature strongly in studies of bio-inspiration, in which scientists investigate the remarkable functions of natural systems and incorporate them into their artificial systems. The growing role of fractals in art suggests that the repeating patterns serve a bio-inspired function beyond the scientific realm - an aesthetic quality. Previous studies demonstrated that exposure to natural scenery can have dramatic, positive consequences for the observer [6-8]. For example, patients recover more rapidly from surgery in hospital rooms with windows overlooking nature. Although pioneering, these demonstrations of 'biophilic' (nature-loving) responses employed vague descriptions for nature's visual properties. Our research builds on these studies by testing a specific hypothesis – that the statistical fractals inherent in natural objects are inducing these striking effects [9, 10].

To quantify the visual intricacy of the statistical fractals, we adopt a parameter employed by mathematicians – the pattern's fractal dimension D [1, 11]. This describes how the patterns occurring at different magnifications combine to build the resulting fractal shape. For a smooth line (containing no fractal structure) D has a value of 1, while for a completely filled area (again containing no fractal structure) its value is 2. However, the repeating patterns of the fractal line cause the line to begin to occupy space. As a consequence, its D value lies between 1 and 2. By increasing the amount of fine structure in the fractal mix of repeating patterns, the line spreads even further across the two-dimensional plane and its D value therefore moves closer to 2.

Figure 3 demonstrates how a fractal's D value has a powerful effect on its visual appearance. This figure summarizes the variety of fractals we have used in our studies, including images from nature, art and mathematics [9, 12-15]. For each of the rows, the image in the left column has a lower D value than that in the right column. Clearly, for the low D fractals, the small content of fine structure builds a very smooth sparse, shape. However, for fractals with D values closer to 2, the larger amount of fine structure builds a shape full of intricate, detailed structure. More specifically, because the D value charts the ratio of fine to coarse structure, it is expected that D will serve as a measure of the visual complexity generated by the repeating patterns. Behavioral research by our group [16] and others [17] confirms that the complexity perceived by observers does indeed increase with the image's D value (Fig. 4).

Returning to Fig.3, the top-row images are photographs of natural scenes (clouds and forests). The second-row images are examples of Jackson Pollock's poured paintings created at different stages in his career [18, 19]. The remaining rows feature different types of computer-generated fractals as follows. The third row shows geographical terrains (in this case viewed from above) and these serve as the source to generate the images below them. To obtain the fourth-row images, a horizontal slice is taken through the terrain at a selected height. Then all of the terrain below this height is colored black and all of the terrain above is colored white. Referred to as the coastline pattern (black being the water), this image is used to generate the fifth-row images by highlighting the coastline edges in white. In the sixth row, grayscale images are generated by assigning grayscale values to the heights of the terrain. Despite their superficial differences in appearance, these 6 families of statistical fractals all possess identical scaling properties and they induce similar effects in the observer. These examples of biophilic fractals differ from the exact fractals shown in the bottom row. Later, we will discuss why these more artificial-looking fractals have a different impact than the biophilic fractals shown above them.

Fractal Fluency

The physical processes that form nature's fractals determine their D values. For example, wave erosion generates the low complexity of the Australian coastline while ice erosion results in the high complexity of the Norwegian fiords. Significantly, although natural objects are quantified by D values across the full range from 1.1 to 1.9, the most prevalent fractals lie in the narrower range of 1.3 to 1.5. For example, many examples of clouds, trees and mountains lie in this range. This forms the basis of our fluency model, which proposes that the visual system has adapted to efficiently process the mid-complexity patterns of these prevalent fractals [9, 12]. We expect this adaption to be evident at many levels of the visual system, ranging from data acquisition by the eye through to the processing of this data in the higher visual areas of the brain.

Based on the phenomenon of synesthesia, in which sensations are transferred between the senses, it is possible that mid-complexity fractals might also hold special significance for tactile and audial experiences in addition to visual ones. This is being tested using three-D printers to generate physical versions of the terrains shown in Fig. 3 and using computers to convert visual stimuli into the sonic equivalents. We also plan to convert the fractals in Pollock's paintings into music and compare people's responses to these equivalent visual and sonic fractals [20].

Our studies of fractal fluency commenced with the eye-motion studies shown in Fig. 5 [9, 11, 21]. The eye-tracking system integrates infra-red and visual camera techniques to determine the eye's gaze when looking at fractal images displayed on a monitor. As expected, the eye motion is composed of long 'saccade' trajectories as the eye jumps between the locations of interest and smaller 'micro-saccades' that occur during the dwell periods. Our results show that the saccade trajectories trace out fractal patterns with D values that are insensitive to the D value of the fractal image being observed. More specifically, the saccade pattern is quantified by $D = 1.4$ even though the viewed image varied over a large range from 1.1 to 1.9. Furthermore, participants with neurological conditions such as Alzheimer's disease exhibited the same fractal gaze dynamics as healthy participants, indicating that the fractal motion is fundamental to eye-movement behavior and is not modified by processing in the higher levels of the visual system [22].

We propose that the purpose of the eye's search through fractal scenery is to confirm its fractal character. If the gaze is directed at just one location, the peripheral vision only has sufficient resolution to detect coarse patterns. Therefore, the gaze shifts position to allow the eye's fovea to detect the fine scale patterns at multiple locations. This allows the eye to experience the coarse and fine scale patterns necessary for confirmation of fractal character. Why, though, does the eye adopt a fractal trajectory when performing this task? We found the answer in studies of animals foraging for food in their natural terrains [23]. Their foraging motions are also fractal. The short trajectories allow the animal to look for food in a small region and then to travel to neighboring regions and then onto regions even further away, allowing searches across multiple size scales. Mathematics shows these fractal searches to be very efficient [11]. This provides the likely explanation for why they are used by animals searching for food and also the eye in its search for visual information [11]. The mid-*D* saccade is optimal for this fractal search because it matches the *D* values found in prevalent fractal scenery. The saccades then have the same amounts of coarse and fine structure as the scenery, allowing the eye to sift through the visual information efficiently.

Effective strategies for processing mid-*D* fractals are also thought be apparent at later stages in the visual system. The brain's visual cortex has been modelled as a set of virtual 'pathways' used to process scenic information [24, 25]. Some pathways are dedicated to analyzing large objects in nature's environment, others to small objects. These pathways have evolved to accommodate fractal scenery as follows. The number of pathways dedicated to each object size is proportional to the number of objects of that size appearing in the scene. In other words, the distribution of processing pathways matches the *D* values that dominate the environment. It has also been proposed that fractal processing utilizes fractal images stored in our memories [26].

Modern neurophysiological techniques such as quantitative EEG (qEEG) and functional MRI (fMRI) offer the potential to refine these preliminary models of how the brain processes fractal scenery. Employing EEG, we use electrodes to measure the time variations in brain activity. Specifically, peaks in 'alpha waves' indicate a wakefully relaxed state while peaks in 'beta waves' are associated with external focus, attention and an alert state [27]. In our studies, $D = 1.3$ fractals are found to induce the largest changes in participants' alpha and beta responses [28]. These changes in alpha waves agree with our skin conductance measurements (Fig. 5), which similarly demonstrate that mid-*D* fractals are stress-reducing [29]. Our preliminary studies using the fMRI technique further indicate that mid-*D* fractals induce distinct responses when compared to those of low or high *D* equivalent images [9].

Enhanced Performance and Fractal Aesthetics

The fluency model proposes that our increased capability to process mid-*D* fractals results in enhanced performances of visual tasks when viewing them [10]. For example, our behavioural studies demonstrate participants' heightened sensitivity to mid-*D* fractals [20]. Using fractal images displayed on a monitor, the pattern contrast was gradually reduced until the monitor displayed uniform luminance. We found that participants were able to detect the mid-*D* fractals for much lower contrasts than the low and high *D* fractals (Fig. 6a) [30]. Similarly, participants displayed a superior ability to distinguish between fractals with different *D* values in the mid-*D* range (Fig. 6b) [30]. Furthermore, the increased beta response in our qEEG studies suggests a heightened ability to concentrate when viewing mid-*D* range fractals [28].

There is also evidence to suggest that pattern recognition capabilities increase for mid-*D* fractals. For example, we are all familiar with imaginary objects induced by clouds. A possible explanation is that our pattern recognition processes are so enhanced by these fractal clouds that the visual system becomes 'trigger happy' and consequently we see patterns that aren't actually there [10]. Our research reveals that mid-*D* fractal images do indeed induce a large number of percepts [31] and that they activate the object perception and recognition areas of the visual cortex [32]. This agrees with our studies of Rorschach ink blots, in which the capacity to perceive shapes in the fractal blots peaks in the lower *D* range [33].

Does fractal fluency also lead to an enhanced processing of visual spatial information and therefore to a superior ability to navigate through environments characterized by mid-*D* fractals? To answer this question, participants navigated an avatar through virtual fractal environments (Fig. 5) [34]. They were instructed to search as quickly as possible for a goal randomly placed within the landscape. In each case, completion speeds and accuracy (the ratio of finding the goal before or after arriving at the distractor) were measured and the overall performance was found to peak at the mid-*D* complexity predicted by the fluency model (Fig. 6c).

All of these enhanced performances raise a crucial question: does fractal fluency also create a unique aesthetic quality because we find mid-*D* fractals relatively easy to process and comprehend? If so, perhaps this 'aesthetic resonance' also induces the state of relaxation indicated by our alpha wave and skin conductance studies? Our behavioral experiments confirm the importance of fractal aesthetics, showing that ninety-five per cent of observers prefer complex fractal images over simple Euclidean ones [35].

Over the past 2 decades, fractal aesthetics experiments performed by ourselves and other groups have shown that preference for mid-*D* fractals is universal in the sense that it is robust to the specific details of how the fractals are generated [12, 30, 36]. Figure 5 shows a participant rating the preference of 2 Pollock paintings with different *D* values displayed on a monitor [12]. Figure 6d shows example results exhibiting the peak in preference, in this case for computer-generated fractals. In addition to these laboratory-based behavioral experiments, a computer server has been used to send screen-savers to a large audience of 5000 people. New fractals were generated by an interactive process between the server and the audience, in which users voted electronically for the images they preferred [37]. In this way, the parameters generating the fractal screen-savers evolved with time, much like a genome, to create the most aesthetically preferred fractals. The results re-enforced the preference for mid-*D* fractals found in the laboratory-based experiments.

Our most recent experiments investigate subtle deviations from this apparent universal preference. Although the population as a whole prefers mid-*D* fractals, Figure 7a highlights 3 sub-groups exhibiting distinct preferences. Whereas the majority's preference peaks at mid-*D*, just under one quarter of the participants are 'sharpies' who prefer high *D* and a similar number are 'smoothies' who prefer low *D* [16]. It will be intriguing to explore the personality traits characterizing these groups. For example, perhaps Autism might be more prevalent in the sharpies group (in which case, fractal stimuli might be useful as a novel predictor of this condition). Alternatively, the *D* value of Pollock's paintings increased as his career progressed, possibly suggesting that creative artists might be drawn to high *D* imagery? Or perhaps his exposure to fractal paintings over the years built up a tolerance for higher complexity so raising his preferred *D* values? Certainly, some of our studies show that urban versus rural living and also age influence fractal preference, indicating that exposure is a factor [38].

Our experiments show that preference for mid-*D* values also breaks down when moving from statistical to exact fractals (Fig. 7b). Given that the fluency model is founded on people's adaption to nature's statistical fractals, it is not surprising that exact fractals induce a different aesthetic impact (indeed, EEG responses were found to dampen when the images were morphed from the statistical to exact versions, emphasizing the adaption of processing fluency to nature's biophilic fractals [39]). Observers are found to prefer higher *D* values for exact fractals, with the peak *D* depending on the specifics of the fractal pattern [14]. For example, Fig.7b shows that the exact fractals of Fig. 3 induce a peak preference in the *D* range from 1.8 to 1.9. This pattern has a high degree of symmetry and it is thought that the associated order increases the observer's tolerance for fractal complexity. For fractals featuring fewer symmetries, the reduced order decreases this tolerance and the preference falls to lower fractal complexities.

This concept of complexity tolerance is further supported by our experiments which project statistical fractal images on walls rather than exhibiting them on computer monitors as done in our previous experiments. The observer then witnesses the fractal pattern embedded within the simplicity of a blank wall. This integration of Euclidean simplicity again increases the tolerance for high fractal complexity and the peak preference rises to higher D values [40].

Tuning the Fractal Aesthetics: The Sinai Light Box

Our on-going studies of fractal aesthetics present an appealing basis for understanding the beauty of nature's scenery. Quantified by D, fractal complexity is a dominant influence on our preferences. Although D values lying between 1.3 and 1.5 represent a magic range for maximizing preference in general, it is also clear that preference can peak outside this range for specific subgroups of observers, and also for subgroups of fractals (e.g. exact fractals) and for situations in which the complexity of the surrounding environment differs from that of the fractal. Based on this diversity of conditions, fractal artists should consider creating art for which D can be adjusted to accommodate for these variations.

The D values of nature's fractal objects are set by the dynamical processes which shape them. For example, the turbulence creating clouds, fissures that shape cracks, and the erosion of coastlines all generate patterns with specific D values. Consequently, once formed, it is rare for natural objects to change their D values. Exceptions include trees, which increase their D values when, each Autumn, the falling leaves expose the higher D fractals of the underlying branches (Fig.8). Another exception involves the foam bubbles shown in Fig. 9. Small bubbles combine to create bigger bubbles, adding larger structure into the fractal mix of the pattern, and this leads to a decrease in D value as a function of time [41].

We aim to outdo nature by building an object which can be used to tune the D value of the fractal pattern to match the observer's preferences. Our apparatus is based on the theoretical research of the Russian mathematician Yakov Sinai. In the 1960s and 1970s, Sinai studied the game of billiards [5]. Figure 10a shows two trajectories of balls bouncing around a standard billiard table. Launching the ball from slightly different locations does do not alter the trajectories in a significant manner. However, the game changes substantially when a circular wall is inserted at the center of the table to create what is now known as a Sinai billiard (Fig. 10b). In his theoretical work, Sinai noticed that the two trajectories then diverge rapidly, ending at significantly different locations on the table. This signature – an extreme sensitivity to initial conditions – is known as chaos. Chaos is prevalent in nature and it is responsible for generating many of the fractals found in our daily scenery. In the case of the Sinai billiard, the outside walls of the table repeatedly reflect the balls onto the curved surface of the inner wall. This curvature causes the trajectories to diverge and induces the chaos. As the balls bounce around the table, their trajectories map out patterns at many scales, gradually building a fractal pattern.

Sinai's billiard is well-known in science as an artificial system in which nature's chaos can be studied (in 2014, Sinai became an Abel Laureate, the mathematical equivalent of a Nobel Laurette, for his work). It has also been used for technological applications. For example, fractal transistors are based on miniature Sinai billiards defined in electronic chips. The enhanced sensitivity of chaotic electricity allows the fractal transistors to out-perform traditional transistors [42-46]. Here, we will exploit Sinai billiards to cross into the world of art and use the chaos to create tunable fractal patterns. To do this, we replace the billiard walls with mirrors and the balls with rays of light. Shown in Fig. 10c, red, green and blue rays of light are shone into three openings in the billiard's corners and the resulting patterns are viewed through the fourth corner either by eye or camera. Figure 10d shows a simulation of the fractal reflections from the spherical surface.

The photographs of Fig. 11 summarize the operation of the actual apparatus, which is comprised of a 30cm wide cube of mirrors, a central spherical mirror and three lamps shining colored light into openings in the upper corners (Fig. 11a). Whereas Fig. 11b shows the resulting pattern from the multiple light rays generated by the three lights (and so matches the simulation shown in Fig. 10d), Figs. 11c,d show the trajectories of individual rays made visible using the fog from dry ice. In particular, Fig. 11c captures the chaos of two rays reflecting off the sphere and Fig. 11d shows the non-chaotic rays when the sphere is removed.

The next step is to investigate how the D values of the fractal light patterns can be tuned. Intriguingly, the impact on D of adjusting the geometric properties of the billiard has not been addressed in previous studies of Sinai billiards. Fig. 12a shows simulations of the patterns generated when the sphere radius is 33% of the box's width and the openings make up 20% of the box's surface area. Increasing the sphere size can be seen to relocate the positions of reflections on the sphere surface (Fig.12b). However, the reflections are relocated in the same fashion irrespective of their sizes. Consequently, the ratio of fine to coarse structure in the fractal pattern is unaltered and so D remains constant. Figure 12c shows the asymmetry introduced when the sphere is moved away from its central position. Again, because this asymmetry is introduced at all scales, the D value remains the same.

The remaining question of adjusting the sizes of the openings produces a much more subtle effect. Widening the openings increases the sizes of the reflections (Fig. 12d). However, the reflections evolve differently at increasingly fine scales. As can be seen in Fig. 13, increasing the openings results in a well-controlled, systematic rise and fall of D (methods for analyzing D values can be found elsewhere [11, 47]). This novel effect is currently being modelled to provide a detailed picture of its origin. However, in essence, widening the openings increases the chance of light rays escaping rather than circulating around the billiard and undergoing multiple reflections. Clearly, mid-sized openings provide the optimal conditions for preserving the rays that generate the smaller reflection patterns, leading to an increase in the ratio of fine to coarse structure in the fractal reflection and a peak in its D value.

This remarkable effect results in our capacity to adjust the fractal patterns based on the observer's aesthetic needs. The current studies purposely considered high fidelity mirrors which minimize any distortions in the reflections. The resulting exact fractals allowed the evolution in D to be demonstrated with clarity. We note that the equivalent statistical fractals can be generated by introducing random bumps into the surface of the spheres.

Conclusion: Fractals as a Bridge between Art and Science

Aesthetics is a rich field for art-science collaboration. In this chapter, we have demonstrated the value of science for understanding a central aspect of art – nature's beauty. In addition to exploring this fundamental question, our fractal studies have important practical consequences. Mid-*D* fractals have the potential to address stress-related illnesses, which currently cost countries such as the US over $300 billion annually.

Our model of fractal fluency also adds fuel to on-going and often controversial discussions within aesthetics studies: to what extent is appreciation driven by the automatic responses of human neurophysiology and biology versus the intellectual and emotional deliberations of the observer? [48] Our studies indicate that a range of automatic processes unfold within a quick time frame. Consequently, we are well on the way to appreciating the fractal object's beauty before we have had time to consciously deliberate on its visual qualities.

In addition to understanding fractal aesthetics, our chapter also considers the role of science in generating fractal aesthetics. Our Sinai billiard was first exhibited at Portland Art Museum in Oregon in 2009 where it was seen by over 60,000 visitors [49]. It was then transferred to Oregon Museum of Science and Industry where it has been enjoyed by countless others. Given its success, it is interesting to consider the Sinai billiard within the spectrum of previous fractal artists. In particular, M.C. Escher and Jackson Pollock present contrasting approaches to their creation of fractals. Escher employed the precision of mathematics to carefully map out his repeating patterns [50]. Pollock, on the other hand, exploited his chaotic body motion to pour his fractals onto a canvas [51, 52]. The Sinai system sits somewhere between the two. Like Pollock, it exploits chaos to effortlessly generate fractals. Certainly, it is impressive that such a simple system – a sphere placed within a cube – could generate such visual complexity. Like Escher, the mathematics of the system can be tuned with precision. By careful adjustment of the openings, the *D* value can be selected.

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Figure Captions

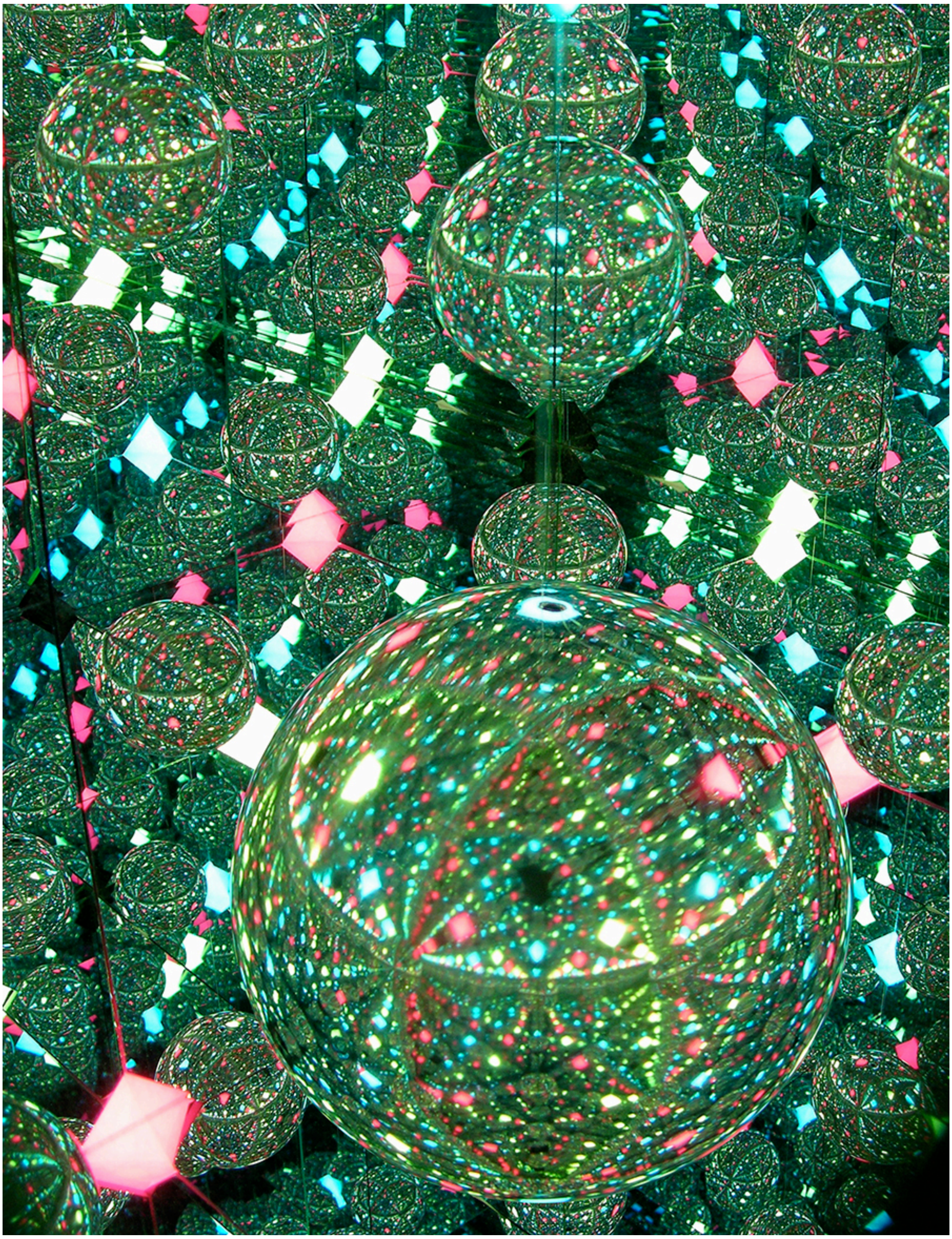

Fig. 1. Red, green and blue light rays reflect off multiple mirrors in the Sinai billiard, building light patterns at many size scales.

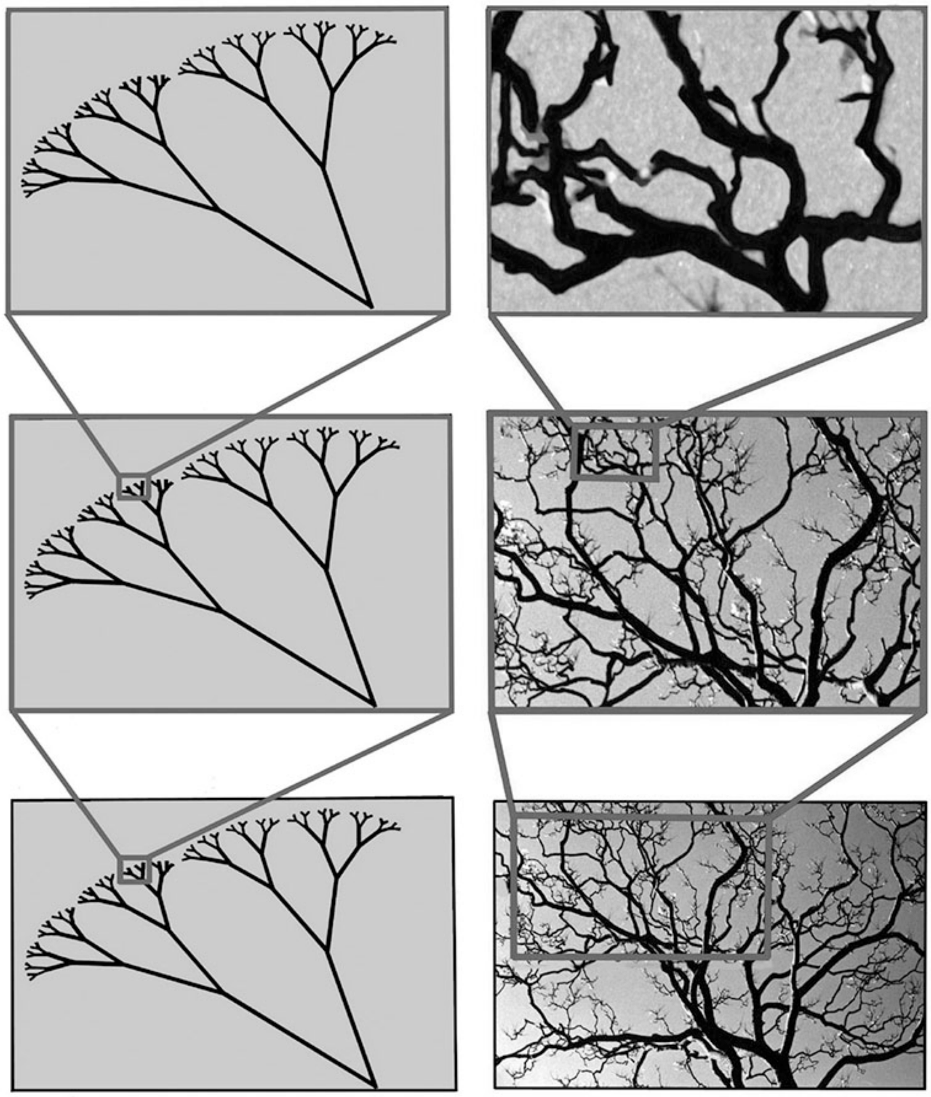

Fig. 2. The branch patterns of an artificial tree repeat exactly at different magnifications (left column). In contrast, only the statistical qualities repeat for a real tree (right column).

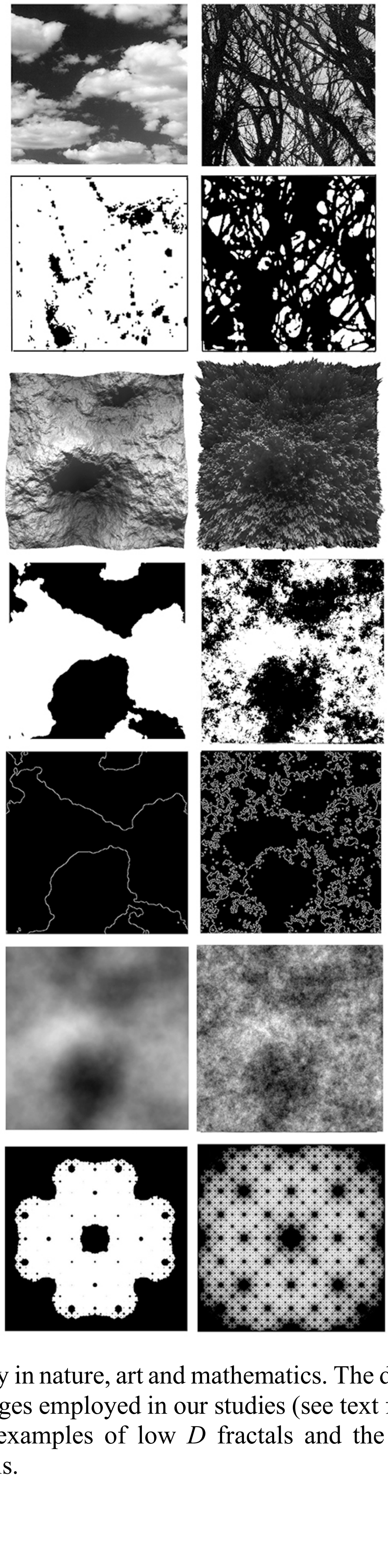

Fig. 3. Fractal complexity in nature, art and mathematics. The different rows summarize the variety of fractal images employed in our studies (see text for details). In each case, the left column shows examples of low D fractals and the right column show the equivalent high D fractals.

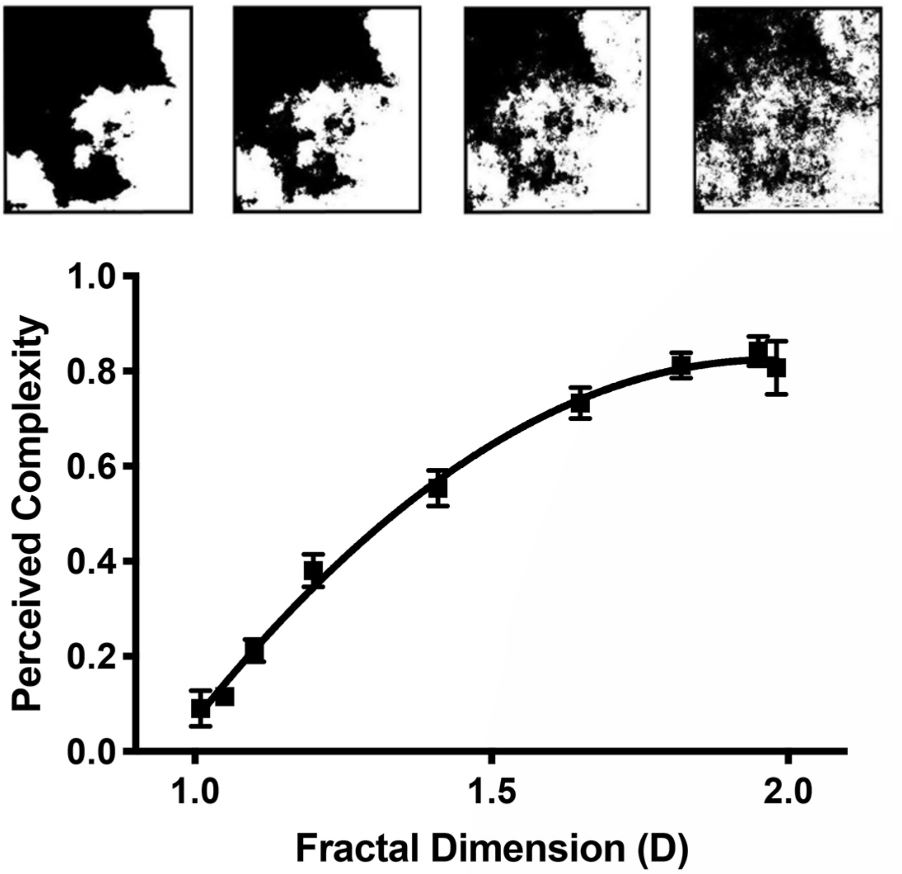

Fig. 4 Perceived complexity increases with the fractal's *D* value. Examples of computer-generated fractals quantified by $D = 1.2$, 1.4, 1.6 and 1.8 are shown above the graph.

Fig. 5 Photographs of some of our behavioral and physiological experiments. Top-left: the eye-tracking apparatus, top-right: skin conductance measurements, bottom-left: fractal scenery displayed on a computer monitor during the navigation experiment, bottom-right: a behavioral preference experiment.

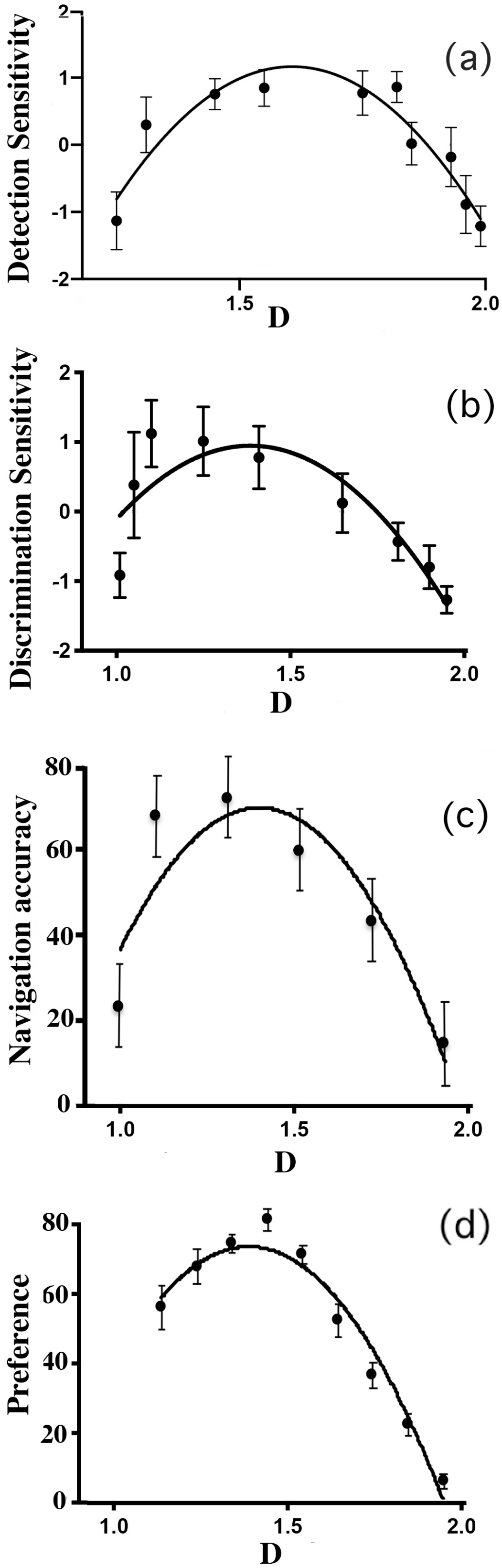

Fig. 6 Capability tasks and preference ratings plotted against the fractal's *D* value. Refer to the individual studies for details of the measurements and the relevant y-axis scale.

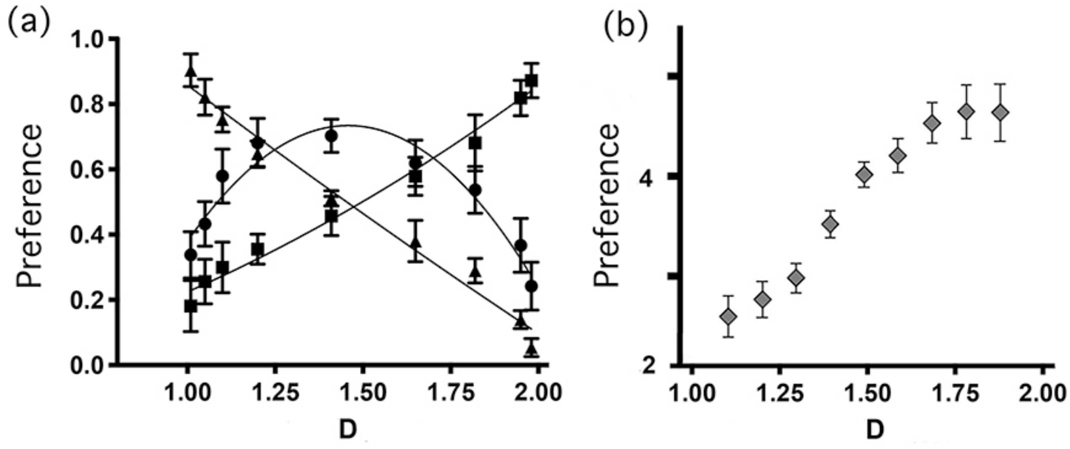

Fig. 7 Deviations away from 'universal' preference behavior. (a) Whereas the majority of participants report a preference peak between $D = 1.3$-1.5 (circular symbols), other subgroups reveal a preference for low D (triangles) and high D (squares) fractals. (b) Exact fractals induce preferences for higher D values than statistical fractals.

Fig. 8 Photographs of trees with (low D) and without (high D) leaves.

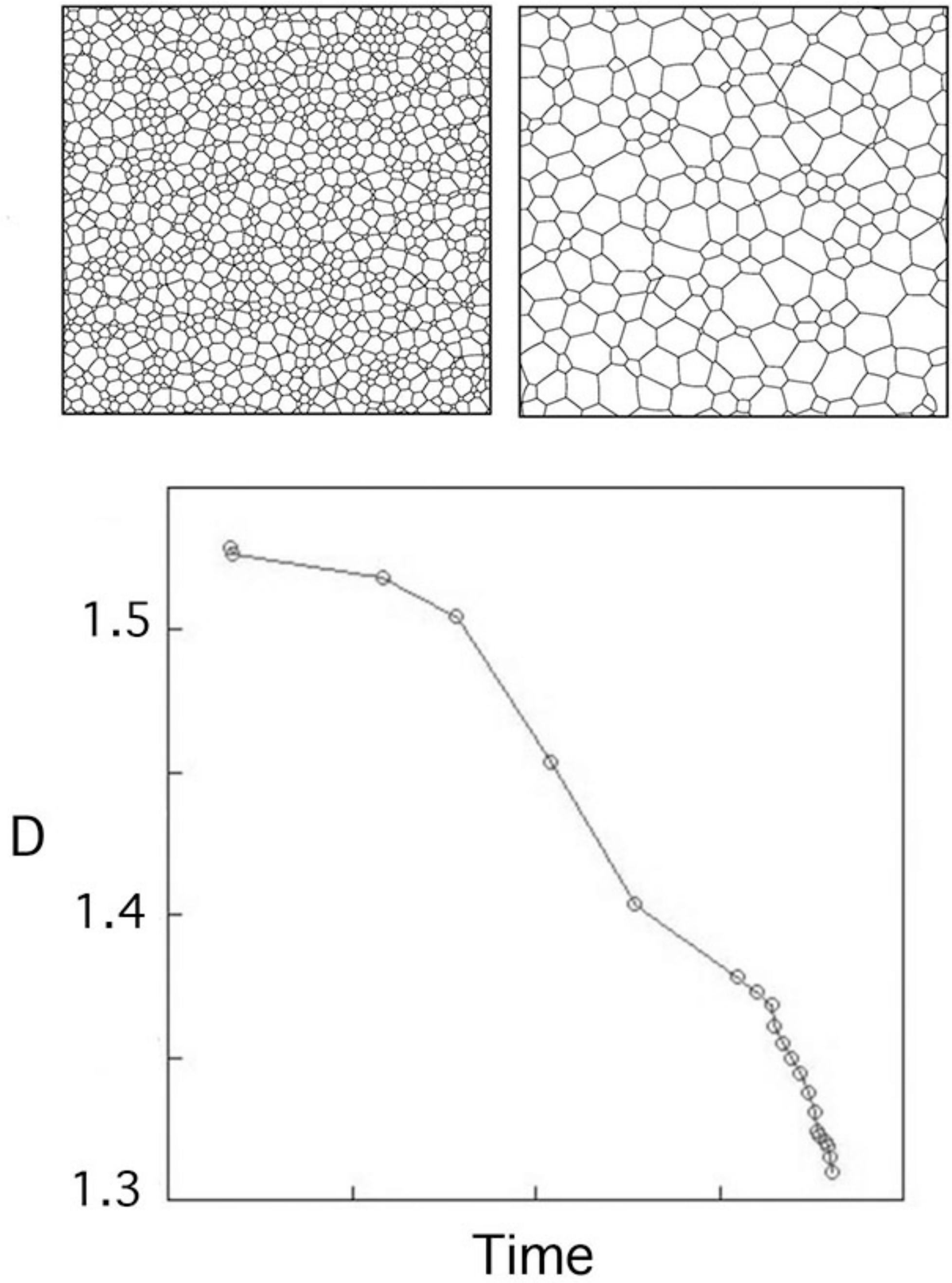

Fig. 9 Top: Computer simulations of bubbles evolving from high *D* (left) to low D (right) patterns. Bottom: A plot of *D* as a function of time.

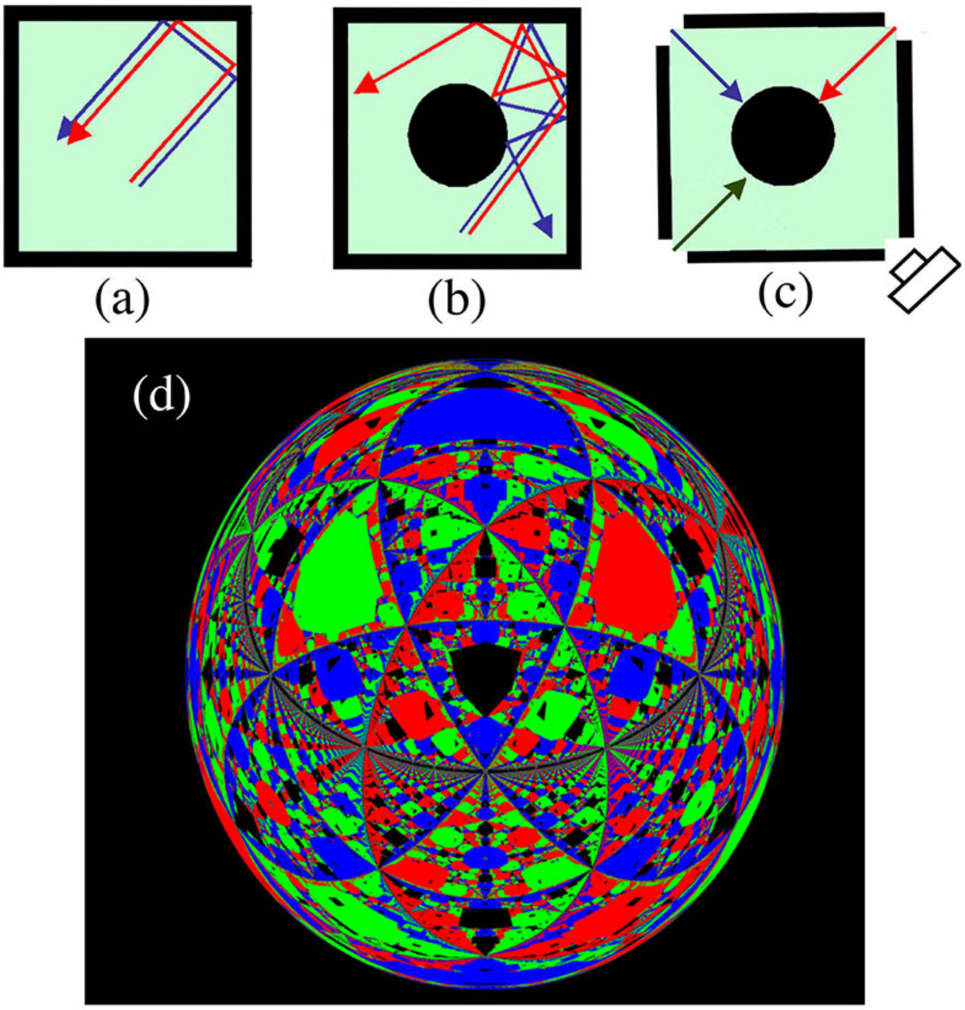

Fig. 10 (a-c) Schematic representations of billiard games. In contrast to the traditional game shown in (a), two balls launched from slightly different locations diverge rapidly for the chaotic game in (b). For the optical game in (c), red, green and blue rays shine in through the openings and a camera takes a photograph through the fourth opening. The simulation (bottom image) reveals reflections on the spherical surface that repeat at multiple size scales.

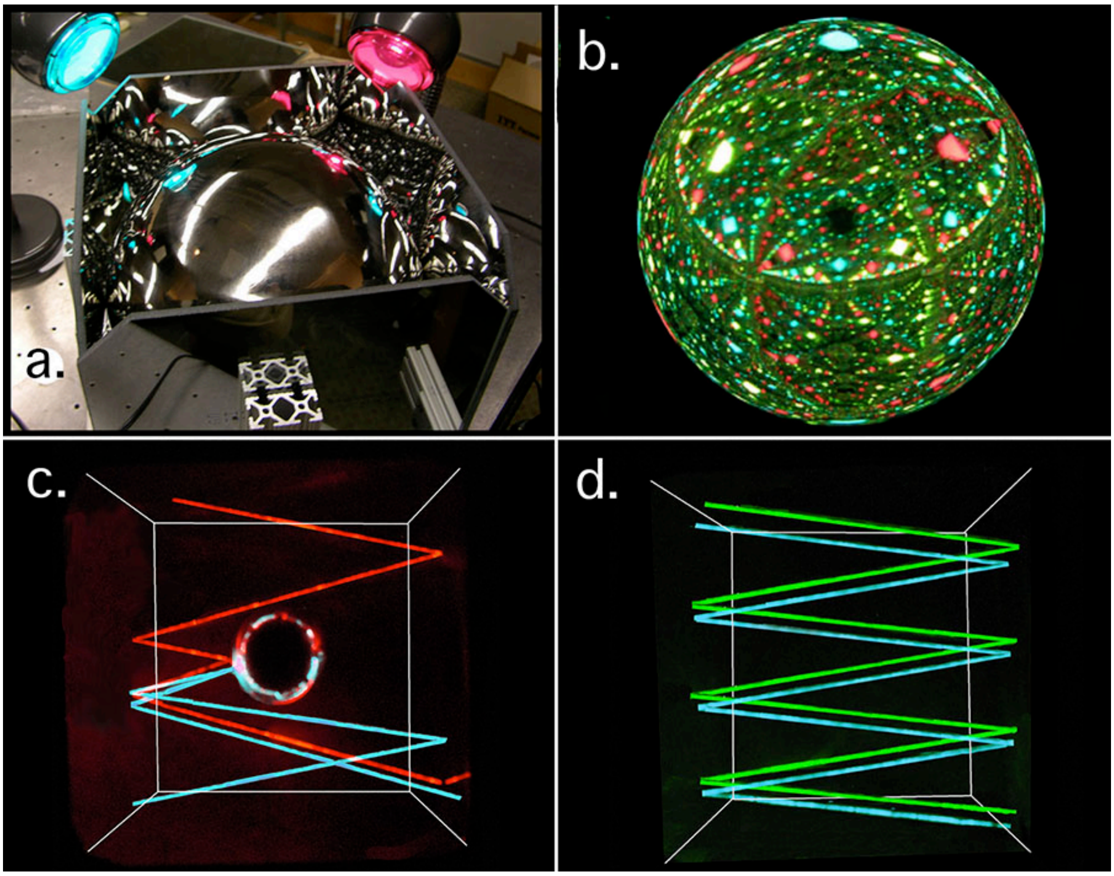

Fig. 11 Photographs of: (a) the optical Sinai billiard, (b) reflected patterns formed on the spherical mirror, (c) two chaotic rays created by reflections off the sphere, (d) two non-chaotic rays occurring when the sphere is removed.

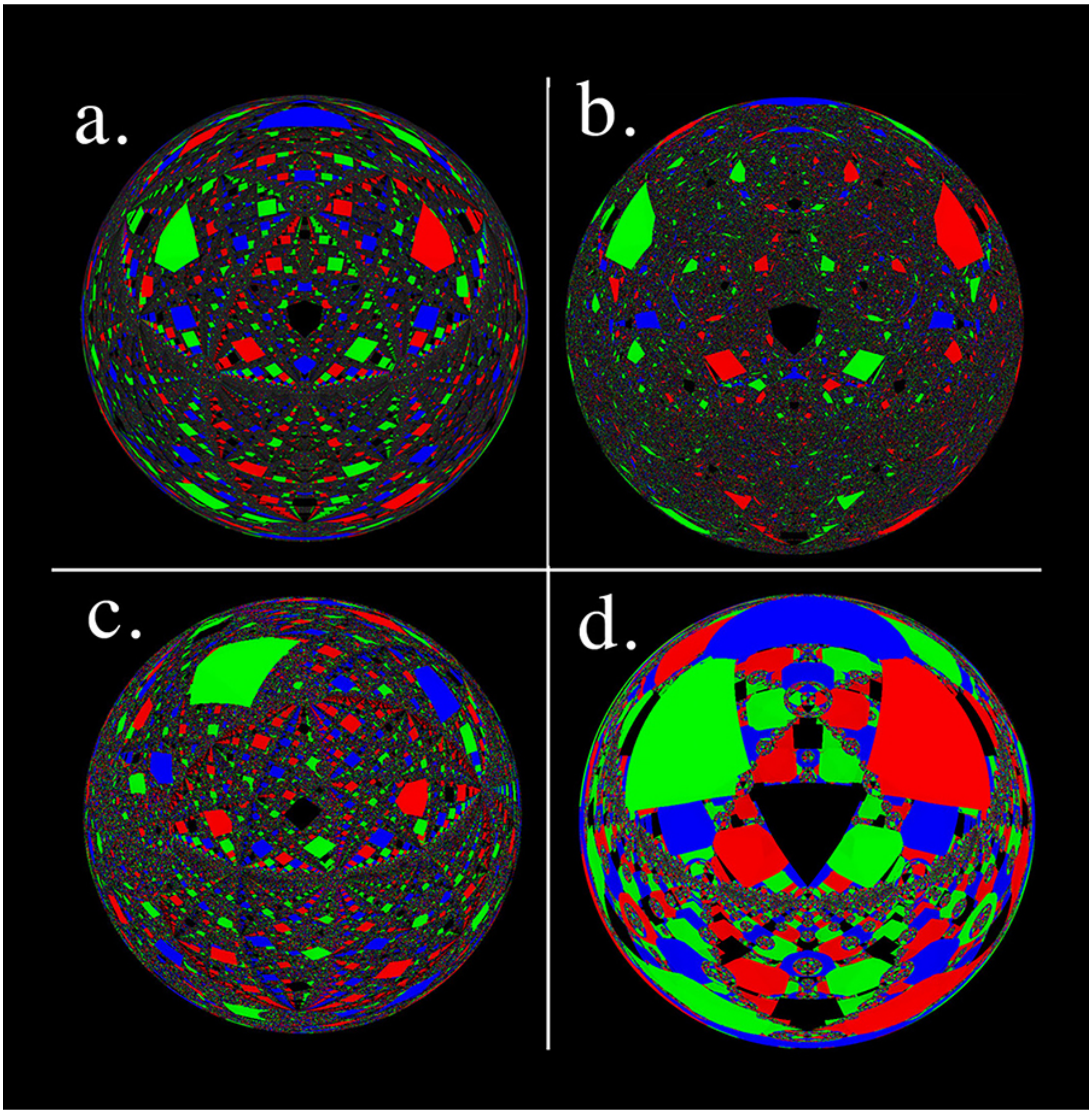

Fig. 12 Simulations of the fractal pattern before (a) and after enlarging the sphere (b), moving the sphere (c), and enlarging the openings (d).

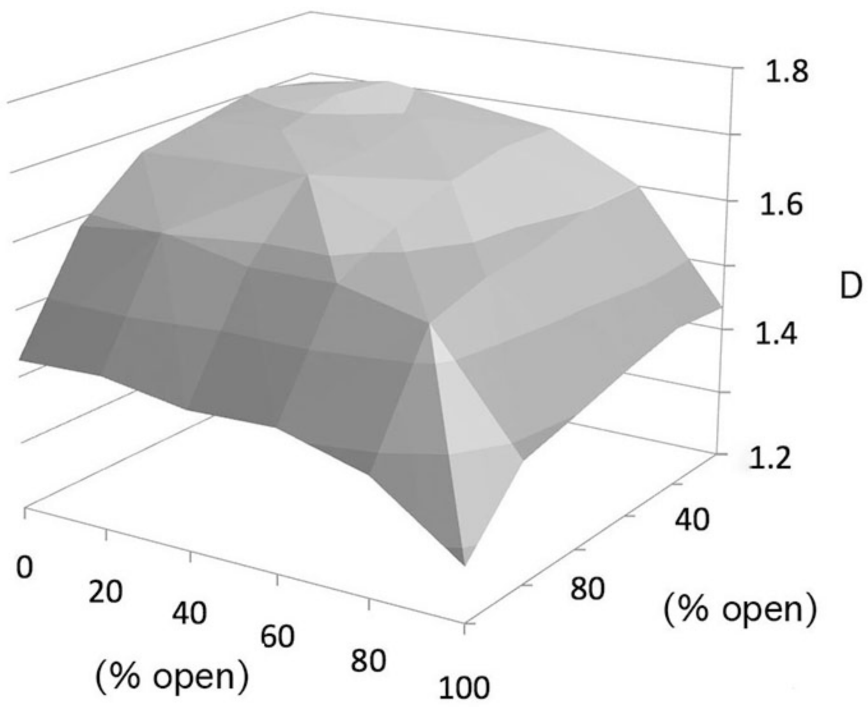

Fig. 13 The fractal pattern's D value plotted as the size of two of the openings are independently increased. The openness is measured as the percentage area of the opening to the sidewall surface area.