Multifractal comparison of the painting techniques of adults and children

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ABSTRACT
Statistical analysis of art, particularly of the abstract genre, is becoming an increasingly important tool for understanding the image creation process. We present a multifractal clustering analysis of non-representational images painted by adults and children using a ‘pouring’ technique. The effective dimensions ($D_0$) are measured for each, as is the associated multifractal depth $\Delta D = D_0 - D_{\infty}$. It is shown that children create paintings whose dimensions $D_0$ are less than those created by adults. The effective dimensions for adult painters tend to cluster around 1.8, while those for children assume typical values of 1.6. In a similar fashion, the multifractal depths for images painted by adults and children show statistically-significant differences in their values. Adult paintings show a relatively shallow depth ($\Delta D \sim 0.02$), while children’s paintings show a much greater depth ($\Delta D \sim 0.1$). Given that the ‘pouring’ technique reflects the body motions of the artist, the results suggest that the differences in the paintings’ fractal characteristics are potential indicators of artist physiology.

Keywords: Multifractal, art, motion

1. INTRODUCTION
The interface between art and science has grown in recent years with the advent of statistical analysis of paintings. Predominantly, attention has been paid to studies of the fractal and multifractal structure of images. These techniques are of particular interest to both the academic and artistic communities, since they serve to identify underlying visual signatures in the art. The fractal dimension can be regarded as a preliminary indicator of information complexity in a pattern: lower fractal dimensions are a measure of shallow complexity, while higher fractal dimensions (i.e. those which approach the dimension of the embedding space) demonstrate high complexity. That is, a line has fractal dimension $D_F = 1$, while a wrapping-curve that densely fills the plane has dimension $D_F \rightarrow 2$. The multifractal spectrum is an infinite family of dimensions $\{D_p = 0, 1, 2 \ldots \}$ that yield key information about the degree to which complexity is manifest in a pattern. Additionally, such analyses can yield important information about how the image was painted, and indirectly give indicators as to whom painted it. As such, fractal analysis techniques have been proposed as part of a greater authentication scheme for known and unknown artworks.

Most notably, the fractal analysis approach was applied by Taylor \textit{et al.} to the abstract expressionist paintings of Jackson Pollock.\textsuperscript{1-3} The analysis was based on the hypothesis that the poured paintings were generated by two physical processes -- the “pour” process (dominating small size scale patterns) and the artist motion process (dominating large size scale patterns). The resulting ‘dimensional interplay analysis’ characterized the paintings in terms of a bi-fractal signature involving two dimensions for the two processes.

The fractal nature of Pollock’s work was confirmed by Mureika \textit{et al.}, who extended the problem to the multifractal arena. In addition to Pollock, this investigation encompassed several paintings by the Québec school \textit{Les Automatistes}.\textsuperscript{4,5} Studies of the fractal dimension of various Pollock images confirmed the results of Taylor’s analysis for the motion-dominated size scales. In addition to examining the multifractal structure of the paint patterns, Mureika \textit{et al.}

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additionally analyzed the luminance gradient patterns, or edges, created by overlapping contrasting colors.\textsuperscript{4,5} Such patterns were termed “perceptual multifractals,” in the sense that identification of the contrast edges mimicked certain processes in the visual cortex. It was determined that while the multifractal spectra of the large-scale patterns did not seem to differentiate between works of different artists, that of the edge patterns did. Mureika has also investigated the dependence of color-space mappings on the fractal analysis.\textsuperscript{6} These initial fractal\textsuperscript{12} and multifractal analyses\textsuperscript{4,6} of Pollock’s work have triggered considerable interest and a number of scaling analysis procedures.\textsuperscript{7,24}

It is important to note that physical fractals, like art or natural objects, have inherent upper and lower cut-offs beyond which their fractal geometry either cannot be resolved or does not exist. For this reason, physical fractals are often referred to as ‘limited range fractals’. A number of factors can influence these cut-offs, the most important generally being the size scale range of the process used to create the physical fractal. The magnification range of nature’s physical fractals can be surprisingly small – the typical range is only 1.25 orders.\textsuperscript{3} For this reason, it is not always clear whether a data trace is actually fractal, or if the data is simply mimicking the fractal scaling behavior for a limited magnification range. Nevertheless, the D values extracted from the multifractal analysis still quantify the associated visual complexities of the pattern. For this reason, we adopt the term “effective dimension” to highlight this ambiguity for limited-range patterns. We stress that the effective dimensions remain important for quantifying the visual characteristics of paintings. For example, previous psychology experiments have demonstrated a correlation between the effective dime

In this paper, we apply the multifractal analysis\textsuperscript{2} to poured paintings by two distinct groups of people – adults (19-20 years) and children (5 years). As a form of ‘action art’, the pouring technique is expected to be closely linked to the motion of the painter. At 5 years, the physiology of motion is not yet mature and therefore the motivation of our study is to examine the relationship between the multifractal spectrum of poured paintings and the age of the painter.

2. MULTIFRACTAL ANALYSIS OF ART

2.1 Multifractal Theory

Fractals are a subset of a larger class of objects known as multifractals. While the former is described by a single scaling dimension $D_f$, the latter is characterized by an infinite number of dimensions $D_i$. $-\infty < q < +\infty$, where $D_q > D_{q+1}$.\textsuperscript{27,28} Multifractal dimensions with $q > 0$ are a measure of the clustering complexity of a set, while those for $q < 0$ describe the anti-clustering behavior. The $q = 1$ multifractal dimension is equivalent to the information dimension, while the correlation (or mass) dimension corresponds to $D_2$. The standard fractal dimension $D_F = D_0$, and if the set is itself a pure or “monofractal” then $D_q = D_0$ for all $q$.

A convenient measure of the complexity depth of a multifractal is defined as\textsuperscript{5}

\[ \Delta D = D_0 - D . \]  

(1)

Patterns with a rich multifractal structure will have large $\Delta D$, while those with little or no variation will show the opposite. A monofractal is characterized by $\Delta D = 0$.

The multifractal spectrum of dimensions is readily calculated by a modified box counting algorithm. The pattern is covered by $N(\varepsilon)$ boxes of scale size $\varepsilon$, of which only $n(\varepsilon)$ actually contain the pattern. These contribute to the multifractal moments,\textsuperscript{27,28}

\[ \Xi(q, \varepsilon) = \sum_{\varepsilon} \left[ p_i(\varepsilon) \right]^q , \quad p_i(\varepsilon) = \frac{n_i(\varepsilon)}{N_{tot}(\varepsilon)} \]  

(2)

where $p_i(\varepsilon)$ is the relative density of the pattern contained in the box. The logarithmic slope $\tau(q)$ of the partition function $Z(q, \varepsilon)$ is related to the multifractal dimension $D_q$ by

\[ $\tau(q) = q \frac{D_q - D_0}{q - q} = \frac{D_0 - D_q}{q} \]  

(3)

where $q$ is the order of the moment.
\[
D_q = \frac{\tau(q)}{q-1}, \quad \tau(q) = \frac{d\log[\Xi(q,\varepsilon)]}{d\log(\varepsilon)}.
\]  

Practical implementation of the modified box counting algorithm generally involves digitization of the image and iteratively binning pixel data into \(N(x_i)\) covering boxes of pixel width \(x_i\) over a finite range of scales. These generally range from roughly the size of the original image, to boxes a few pixels in width. Numerical estimates of fractal and multifractal dimensions are then obtained by linear regression of the partition function values (2) as a function of scale size.

2.2 Painting Analysis

For our study, we recruited 18 children and 34 adults and supplied them with identical paint surfaces (sheets of paper with dimensions of 61.6 cm by 95.5 cm) and identical synthetic paint diluted to a common fluidity suitable for pouring. For simplicity of interpretation and analysis, the study considered monochromatic paintings. Two representative paintings are shown in Figure 1.

![Figure 1. Sample of child (left) and adult (right) poured abstract paintings.](image)

The paintings were scanned to generate images of 2880×2038 pixels (adults) and 2880×1936 pixels (children). The box-counting technique was carried out over the range of covering box sides \(x = 2048\) pixels through to \(x = 4\) pixels. The dimensions \(D_0\) and \(D_{50}\) are calculated by linear regression on the box counting data, in order to provide a suitable estimate of the multifractal depth (Equation 3). The linear fit considers box sides in the range \(x = [8,1024]\). The paintings considered herein are consistent with the bi-fractal behavior observed in previous research, i.e. with a knee between “pour” dominated and “motion” dominated painting mechanisms. For the purposes of this paper, we are interested in the motion-dominated regime.
The analysis results are shown in the binned histograms of Figure 2. The adult paintings show dimensions with average dimensions of $D_0 = 1.86$ and $D_{50} = 1.82$, while those for the children’s paintings have lower means ($D_0 = 1.65$, $D_{50} = 1.54$) and a wider spread. The corresponding mean fractal depths are $\Delta D = 0.04$ for adults and $\Delta D = 0.11$ for children. These dimensions capture the visual complexity of the associated paintings. Consider the two representative paintings shown in Figure 1. A higher $D_0$ value indicates a larger number of occupied boxes at small box sizes. Thus a striking visual consequence of a large $D_0$ is that the larger contribution of fine scale structure generates a spatially dense pattern, and this can be seen in Figure 1 for the adults. Similarly, the larger depth revealed by the multifractal analysis can be seen in the reduced spatial uniformity of the childrens’ paintings.

2.3 Comparison to other motion mechanisms used to generate poured paintings

To explore this result further, we considered a poured painting of the Surrealist painter, Max Ernst. Ernst used a pendulum featuring a container that poured paint onto a canvas laid out horizontally on the ground below the pendulum. He guided the pendulum’s motion using his hands and generated works such as “Young Man Intrigued by the Flight of a non-Euclidean Fly” (1942). He was hoping to tap into his subconscious by using the resulting paint trajectories as a springboard for free association. For example, he perceived the image of a face in the paint trajectories of “Young Man Intrigued by the Flight of a non-Euclidean Fly” and developed the painting further by sketching in a face. For the purposes of our study, we are interested in the underlying paint trajectories and have therefore electronically extracted these trajectories – shown in Figure 3.

We applied the multifractal analysis to Ernst painting, and the results are indicated by arrows in the histograms of Figure 2. Since Ernst used the pendulum to pour his patterns, we hypothesized that this action would suppress some of the physiological motions associated with the adult paintings, and that, as a consequence, his paintings would more closely resemble those of the children. Indeed, the effective dimension $D_0$ of his painting falls in the range defined by the children’s paintings. Intriguingly, however, the multifractal depth is more consistent with the mean of the adults’ paintings. We emphasize the preliminary nature of the result – the multifractal analysis been applied to just one of Ernst’s paintings. Nevertheless, it represents an additional insight into the fascinating connection between the motion of a painter and the resulting fractal characteristics of the painting. Future investigations will use infrared motion capture techniques to facilitate a multifractal analysis of an artist’s motions as well as the painting that results from those motions.
3. CONCLUSIONS

In this paper, we have applied a multifractal analysis to the poured paintings by two distinct groups of artists – adults and children. Since children and adults differ dramatically in degrees of motor control, we hypothesized that the effective dimension and multifractal depth could serve as signatures of such physiological differences. Our results show a clear distinction between the two groups. This apparent connection between a painting’s fractal properties and the maturity of the painter will be expanded upon in an upcoming manuscript. We hope that the initial results presented in this paper will serve as motivation for other researchers to adopt pattern analysis techniques and explore the link between the visual characteristics of action art and the physiology of the artist’s motion.

ACKNOWLEDGEMENTS

During this research, M. S. Fairbanks was supported by an NSF IGERT and R. P. Taylor was a Research Corporation Cottrell Scholar.

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