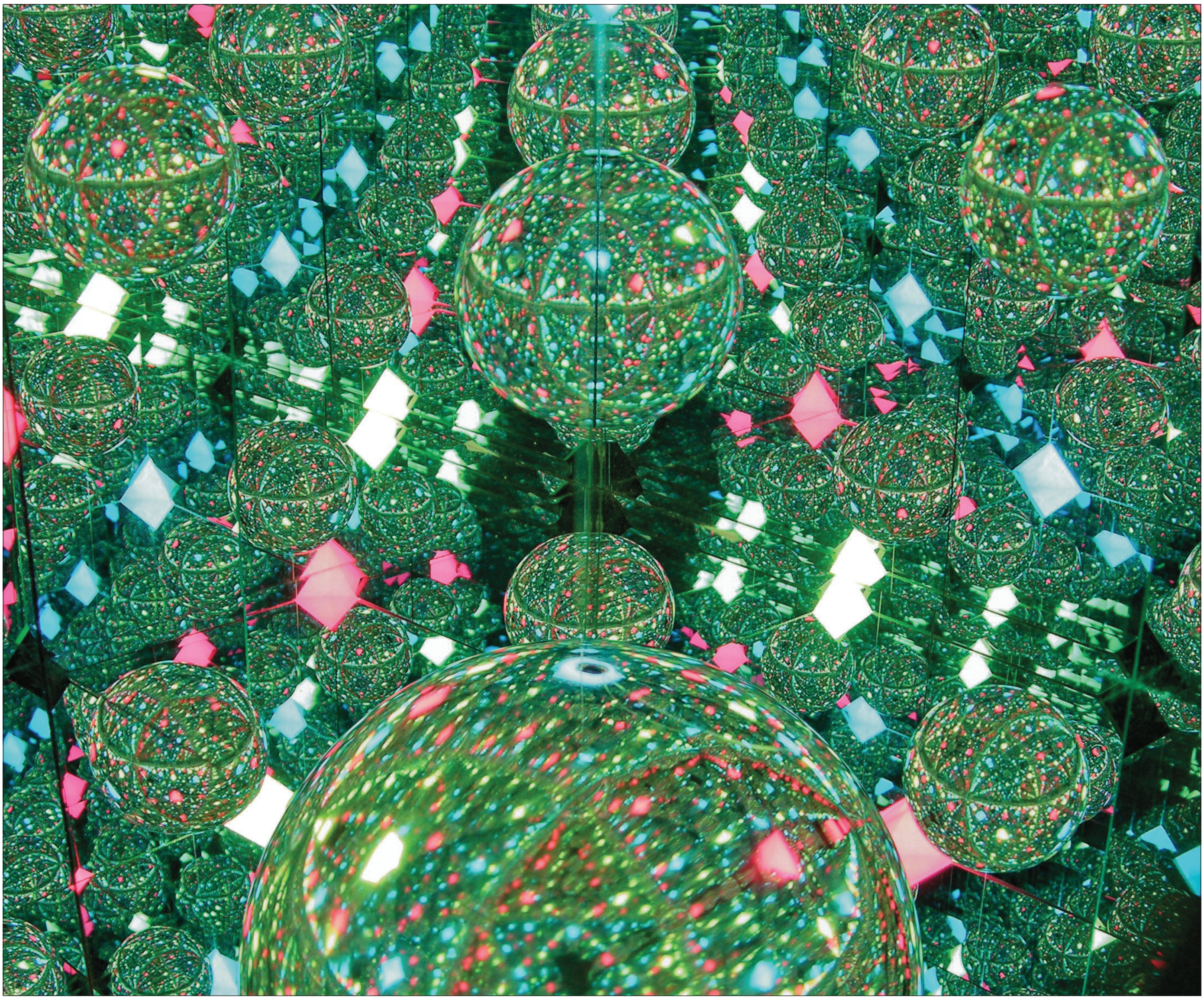
A fascination with fractals



Richard Taylor

Richard Taylor describes how art and science are intertwined through a shared endeavour to understand nature's chaos

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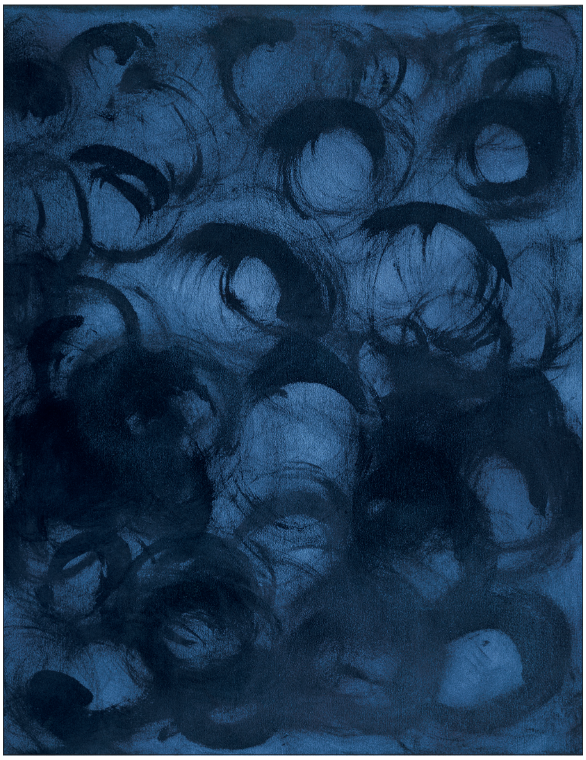
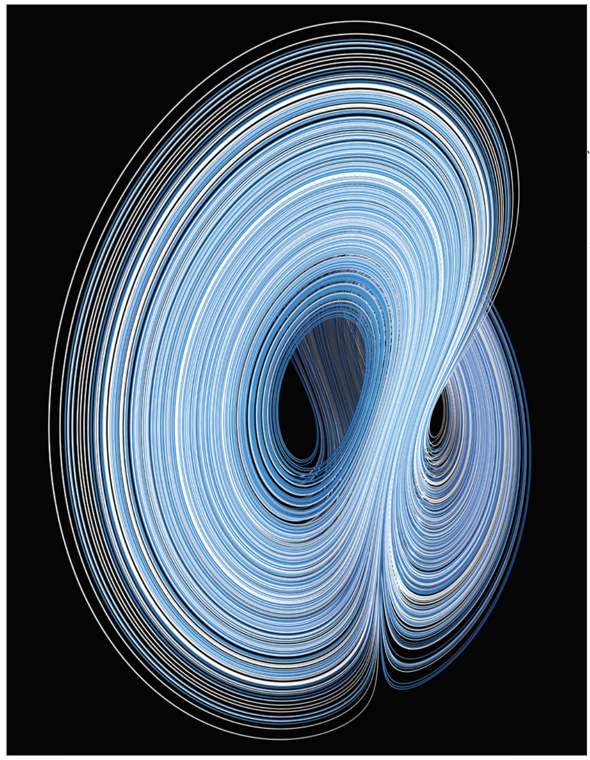
It was as a PhD student when my fascination with fractals began. Studying physics at the University of Nottingham back in the mid-1980s, I had started to form ideas about how Jackson Pollock's paintings were fractal. But in those days interdisciplinary research between science and the arts had not really taken hold, so this side project of mine had to be a covert operation. I would get books out of the science library and sneak them into the arts library so I could lay both patterns – photographs of nature's fractals and Pollock's paintings – on a table in front of me to compare them. When the arts library staff found me with a science book, they were not pleased. In my fiery young days I accused them of being the "culture police". It was a lot of fun.

The appeal of art and science's affinities stayed with me throughout my PhD. To break the boredom of working late at night in the physics department, I would wander down to the laboratory of Peter Mansfield, who would later share the 2003 Nobel Prize in Physiology or Medicine for his development of magnetic resonance imaging (MRI). His group used to keep a big rubbish bin out in the corridor and I would search through it to find Polaroids of his MRI scans, which were mainly images of fruit at that stage. I would take the photos home and draw them. Eventually, I rented space with some artists in an old warehouse, where we would create huge abstract paintings based on Mansfield's images.

Fast-forward to the present day and I am



Chaos in art Yves Klein's 1960 artwork "Vent Paris-Nice" (COS 10) was created by exposing a freshly painted canvas to the weather on a drive from Paris to Nice. This was part of his series of "Cosmogonies", which used weather, river water, plants and soil to create art from the chaos of nature.



Chaos in science Mathematical artwork based on the Lorenz atmospheric model. If you consider two almost identical atmospheres, the two will quickly become very different. However, even if they follow paths that seem unpredictable, they all accumulate on the same object shaped like a butterfly, a strange attractor.

now a professor of physics, psychology and art at the University of Oregon. I was lucky enough, in 2002, to receive a five-year fellowship from the Research Corporation for Science Advancement to explore, on a more formal basis, my interests in the relationship between fractals and science. As a result of this intellectual freedom, I was able to discover, through two recent collaborations with psychologists and physicists, the fractal nature of human balance, and also the fractal beauty of a 1970s mathematical model made real. What both of these examples share is one simple but profound underlying factor: chaos.

Seeds of chaos

Despite its ubiquity in nature, the signature of chaos – an extreme sensitivity to initial conditions – was not directly observed until the early 1960s. Curiously, it became apparent in both art and science at the same time. And what would drive these simultaneous developments? That common force of nature, the weather.

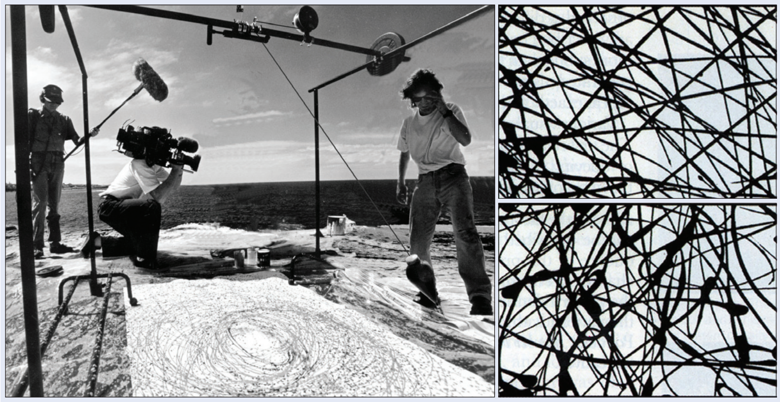
Curiously, chaos became apparent in both art and science at the same time, driven by that common force of nature, the weather

On a blustery Boston morning in 1961, American meteorologist Edward Lorenz set about his weather-forecasting research, which involved some simple models, a cutting-edge computer and some data sets to analyse. The data he had was accurate to six decimal places, but on this morning he decided to punch in only three decimal places for each value. Returning an hour later after a coffee break, Lorenz was puzzled by the data. In no way did it resemble his previous results, and his first thought was that he had better replace one of the vacuum tubes in his computer. But he then realized that the wildly unexpected output was down to those three missing digits – the slight change in initial conditions had led to a completely different result.

The previous year, the French abstract artist Yves Klein had encountered chaos via a different route. Reports on this tale differ and are perhaps apocryphal, but the story goes that one day Klein was due to deliver a painting to a gallery several hours' drive away, and in the morning had not even made a start. Klein's agent, becoming increasingly worried, reminded him of the looming deadline and the agreed subject: patterns in nature. Klein kept his cool, strapped a canvas to the roof of his car and drove to the gallery through a savage storm. On arrival, Klein explained to the gallery owner that they were now privileged to possess an artwork composed not by him, but by nature.

Analogous to a scientific experiment, Klein had used the canvas as a device to record the patterns of dynamic processes and, in doing so, went down in history as one of several artists of this era to experiment with the physical processes that generate abstract art. Another key figure in this effort was the German artist Max Ernst, who in the 1940s had invented the guided pendulum. It was another chaotic process, but this time involving human motion. "Tie a piece of string, one or two metres long, to an empty tin can, drill a small hole in the bottom and fill the tin with fluid paint," he wrote. "Then lay the canvas flat on the floor and swing the tin backwards and forwards over it, guiding it with movements of your hands, arms, shoulder and your whole body. In this way surprising lines drip onto the canvas." By holding the tin in his hands and using his body to guide the pendulum, the paint trajectories recorded the swaying motions of his human balancing act. Jackson Pollock, who was Ernst's contemporary, similarly poured paint onto vast, horizontal canvases.

Kicking off chaos



R P Taylor *et al.* 2011 *Front. Hum. Neurosci.* **5** 60

Left: The author demonstrates a kicked pendulum (see p41) for ABC Television. Right: Examples of non-chaotic (top) and chaotic (bottom) patterns produced using the pendulum.

Other artists followed suit, with William Green dragging bicycles across painted canvases in 1958 and Niki de Saint Phalle firing bullets at bags full of paint to produce splatter patterns in 1961. In the same year as his rainstorm piece, Klein used paint to record the motions of other physical processes. In one controversial piece he smeared paint on nude women and then pulled them across canvases to celebrate the gestural marks produced by the writhing motions of their bodies.

The art critic Harold Rosenberg introduced the term "action art" to describe these performances. He declared, "What was to go on canvas was not a picture but an event. The painter no longer approached his easel with an image in mind; he went up to it with material in his hand to do something to that other piece of material in front of him. The image would be the result of this encounter."

Psychologist Rudolf Arnheim was, however, unimpressed with the patterns generated by action art, assuming the unpredictability originated from random, and therefore meaningless, actions. "Nothing predictable seems to remain," he said.

In reality, these patterns were the signature of a novel form of order, the unpredictability of which simply masqueraded as disorder. This new order would slowly gather attention within the scientific community and become known as chaos.

The chaotic journey

The first person to spot chaos's calling card – unpredictability in the absence of randomness – was French mathematician Henri Poincaré in the 1880s. At the time he was grappling with the extreme sensitivity of a three-body system when applying Newton's laws to planetary motion. "It may happen that small differences in the initial conditions produce very great ones in the final phenomena," he observed. "Predictability becomes impossible."

The nonlinear equations that Lorenz published in 1963, following his discovery on that wintry Boston morning, were similarly sensitive – they amplified small differences in initial conditions, leading to long-term unpredictability (*J. Atmos. Sci.* **20** 130). His results demonstrated the dramatic impact of chaos on people's daily lives: forecasting the evolution of natural phenomena such as the weather is fundamentally unreliable.

Following his 1972 talk "Predictability: Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?", Lorenz's discovery became known as the butterfly effect. The popular term chaos was in fact only adopted after it was used in the title of a 1975 article by mathematicians James Yorke and Tien-Yien Li.

Along with its naming, the discovery and evolution of chaos seem to be as chaotic as the phenomenon itself. The development of fractals – the geometry describing the patterns left behind by chaos – was equally precarious. The first article to hint at nature's fractal objects was published in 1961 – the year of Lorenz's famous coffee break. However, it received little attention because its author, British mathematician Lewis Fry Richardson, had died eight years earlier. Furthermore, its title – *An Appendix to the Statistics of Deadly Quarrels* – did not offer clues to the secrets of nature that lay within.

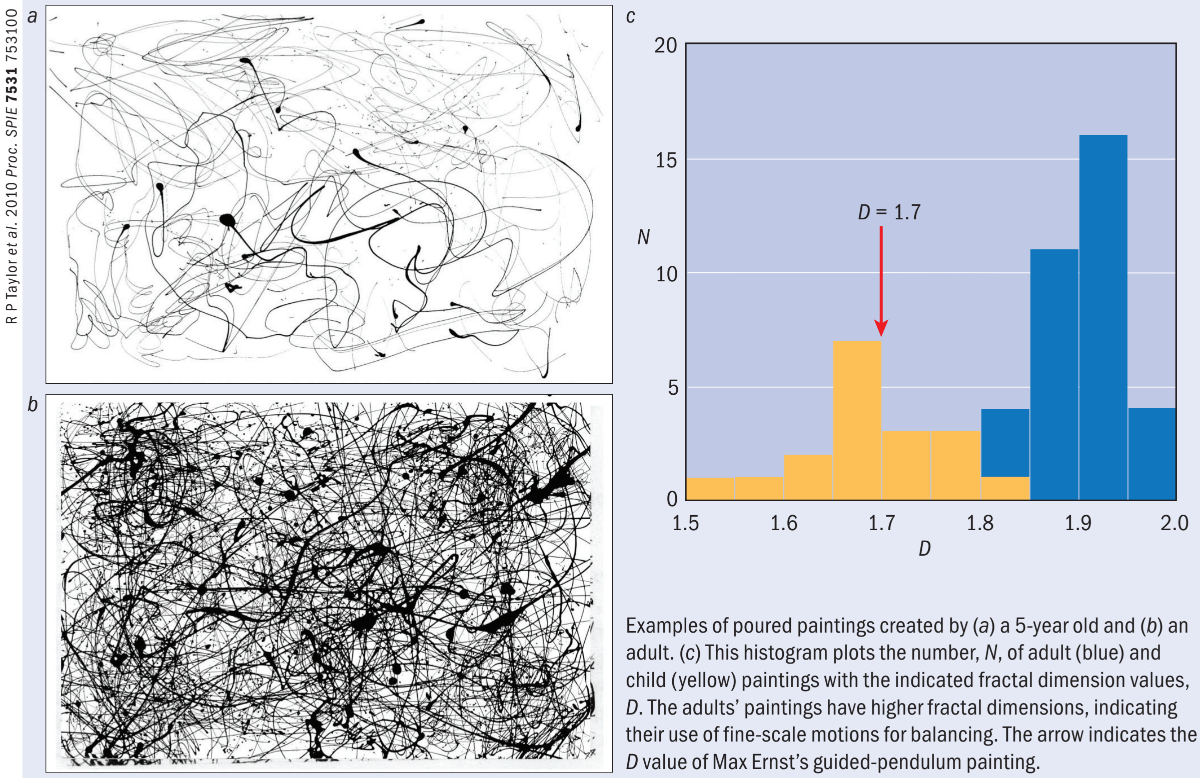
Richardson – who was a pacifist – had been investigating the length of the British coastline in the hope of developing a mathematical theory to explain the likelihood of countries going to war. However, efforts to prove his hypothesis – that countries with long boundaries are more likely to fight – were frustrated by the fact that the boundary lengths quoted in the literature varied with measurement resolution.

After discovering Richardson's findings, French–Polish mathematician Benoit Mandelbrot fired new life into them within the context of fractal geometry. In 1967 he explained the resolution dependence by picturing the coastline as a set of patterns recurring at increasingly fine scales. This revelation triggered Mandelbrot to search through other structures found in natural scenery, culminating in his influential book *The Fractal Geometry of Nature*, published in 1982. During this time he also explored the mathematical construction of fractal images and generated the now-famous Mandelbrot set.

Balancing act

Some 50 years on from Lorenz's publication, the impact of chaos and fractals has spread from mathe-

1 Age advantage



Examples of poured paintings created by (*a*) a 5-year old and (*b*) an adult. (*c*) This histogram plots the number, *N*, of adult (blue) and child (yellow) paintings with the indicated fractal dimension values, *D*. The adults' paintings have higher fractal dimensions, indicating their use of fine-scale motions for balancing. The arrow indicates the *D* value of Max Ernst's guided-pendulum painting.

matics and the physical sciences to human physiology and psychology. Many human physiological processes, after all, exhibit chaotic and fractal characteristics, ranging from excited neuronal networks to heartbeats and breathing patterns. Physical motions, such as variations in stride lengths when walking and the swaying motions when trying to balance, are also fractal.

In collaboration with US physicists Jonas Mureika of Loyola Marymount University and Matthew Fairbanks of the California Maritime Academy, I have studied the fractal characteristics of balance motion by analysing paintings created by adults and five-year-olds (figure 1). As with Ernst's guided pendulum, the participants held the tin in their hands and poured paint onto the horizontal canvas. However, in our study the tin was not attached to a pendulum, allowing a freer, more natural balancing motion as the adults and children leaned over the canvas to paint. Computer analysis of the paintings quantified the fractal dimension, *D*, of the resulting patterns.

The fractal dimension characterizes the relative contributions of coarse and fine structure in fractal patterns, where fine structure plays a more dominant role in high-*D* patterns. This parameter also relates to the pattern someone produces when pouring paint – where wobbles relating to their balance translate directly onto the canvas. Someone who makes many swift, fine-scale changes in order to balance will produce a pattern with many small trajectories of paint; someone who takes longer to correct their balance will move further before changing their direction, and will generate long, looping trajectories on the canvas.

Many human physiological processes exhibit chaotic and fractal characteristics, ranging from excited neuronal networks to heartbeats and breathing patterns

The children's paintings had lower *D* values than those of adults (figure 1*c*). Because children's balance is not fully developed, this result confirmed our hypothesis that fine-scale motions are crucial for balancing.

My collaborations with psychologist Branka Spehar of the University of New South Wales in Australia reveal that people prefer to look at fractals with *D* values in the 1.3–1.5 range, indicating that the children's paintings are more aesthetically pleasing than those of the adults. Many of nature's fractal objects have *D* values in this range, offering a possible explanation for the therapeutic value of staring at certain clouds, mountains and forests. For this reason, these mid-*D* patterns are known as biophilic fractals, and our physiological measurements show they reduce the observer's stress levels by up to 60%.

Intriguingly, Ernst's guided pendulum generated $D = 1.7$ patterns. Although his art is therefore more aesthetic than that generated by most adults, who typically created $D = 1.9$ patterns, the chaos driving Ernst's physiology prevented him from creating fractals in the lower-*D* aesthetic range.

Mathematics' helping hand

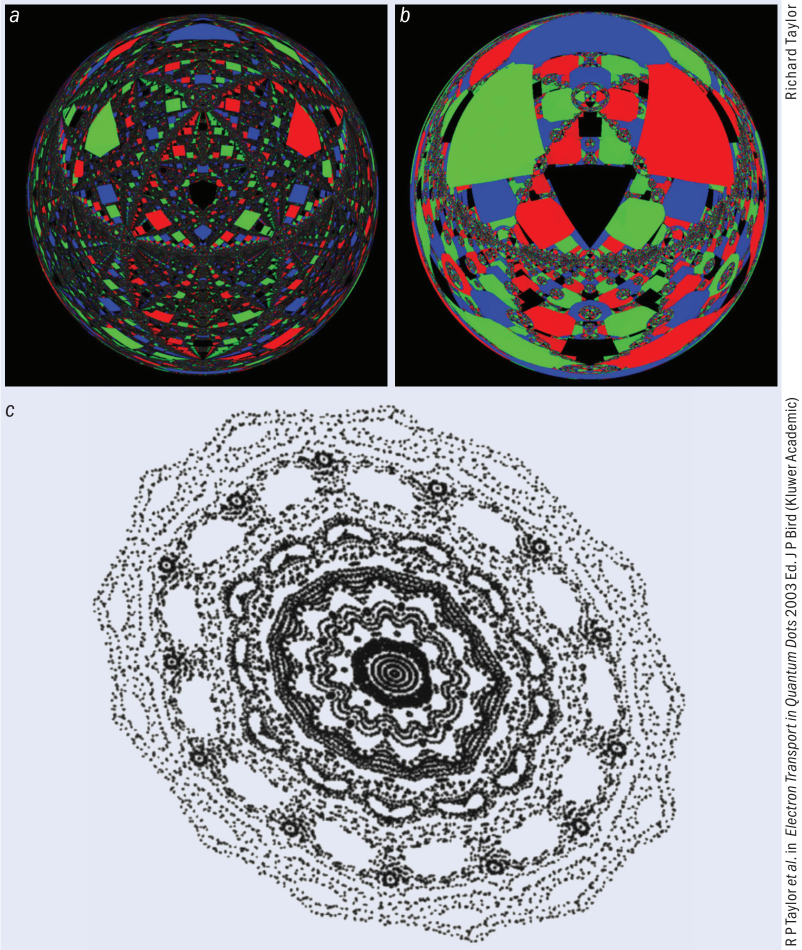
There is a way to get around the physiological constraints of creating high-*D* fractals: mathematics. Mathematicians have generated fractals for more than 150 years. Karl Weierstrass (in 1861), Georg Cantor (in 1883), Giuseppe Peano (in 1890), David Hilbert (in 1891), Helge von Koch (in 1904) and Wacław Sierpiński (in 1915) all created famous fractals. They were generated using two simple instructions: one rule that describes the basic pattern (the seed) and another rule (the generator) that explains how to repeat this basic pattern.

Mandelbrot introduced the term fractal in 1975 to unite the scale-invariant properties he and Richardson had discovered in natural structures with those of the previous mathematical patterns. The earlier mathematicians had failed to spot the similarity between their patterns and those of nature. "Nature has played a joke on mathematicians," stated physicist Freeman Dyson, soon after Mandelbrot's discovery. "The 19th-century mathematicians may have been lacking in imagination, but nature was not. The same pathological structures that the mathematicians invented to break loose from 19th-century naturalism turn out to be inherent in familiar objects all around us."

Although the seed-generator technique allows the *D* value to be tuned with precision, Mandelbrot called these patterns "cold and dry" when compared with nature's shapes. In contrast, Poincaré investigated fractals that emerged spontaneously from the equations of nature's chaos. In the 1970s the Russian chaologist Yakov Sinai demonstrated the simplicity of this approach by considering the game of billiards. In his mathematical models, he placed a fixed circle in the centre of a 2D table and aimed "balls" at it. He showed that the balls' motion around the table was chaotic.

In collaboration with US physicists Ben Van Dusen of the University of Colorado and Billy Scannell of the University of Oregon, I constructed a physical version of Sinai's game. Because it is hard to follow the trajectory patterns of the balls as they scatter rapidly around the table, we replaced the walls with mirrors and used light rays instead of balls. We then photographed the emerging fractal patterns of light (see image on p37). By adjusting the size of the openings, we tuned the escape rate of the trajectories. The corresponding change in chaotic dynamics induced a shift in *D* towards the aesthetically preferred 1.3–1.5 range (figure 2*a* and *b*).

2 Chaotic beauty



Ray-tracing simulations of (*a*) high-*D* and (*b*) low-*D* fractal patterns generated by tuning the size of the openings in Sinai's game, in which billiard balls are aimed at a fixed circle in the centre of a table. (c) Plotting the ensuing chaotic motion of the simulated trajectories in phase space as velocity (*y*-axis) against position (*x*-axis) reveals the beauty of the resulting Poincaré plot.

Kicking the pendulum

Sinai's game serves as a modern example of action art. The apparatus sets a chaotic process in action and records the patterns generated. Kicked pendulums do the same thing. They descend into chaos when knocked at a frequency slightly lower than the one at which they naturally swing. Unlike Ernst's guided pendulum, the nonlinear motion of a kicked pendulum can be manipulated by changing the frequency and size of the kick applied to it. This unpredictable motion induced by the pendulum has become a standard system for demonstrating chaotic trajectories. The spatial patterns of paintings can evolve by tuning the pendulum's kick.

The aesthetics of action art can be further appreciated by displaying the trajectories in phase space. Figure 2*c* shows a "Poincaré plot" for Sinai billiard trajectories simulated in a collaboration with physicist Mark Fromhold at the University of Nottingham. The equations describing the kicked pendulum and Lorenz's weather dynamics also reveal intricate fractal patterns in their phase-space plots.

Mathematician David Ruelle coined the term "strange attractor" in 1980 to convey some of the exotic shapes that emerge in phase-space diagrams. "These systems of curves, these clouds of points," he wrote, "suggest sometimes fireworks of galaxies, sometimes strange and disquieting vegetal proliferations. A realm lies here to explore and harmonies to discover."

In 1959 physicist-turned-novelist CP Snow warned of the growing rift between the arts and sciences in his influential lecture "The two cultures". In my experience, most people misinterpret Snow's treatise as a declaration that this rift is natural and therefore inevitable. In reality, he was highlighting the need for common language across the arts and sciences to defeat the rift. In my own career, the common language of fractals has allowed me to weave chaotically in and out of art school and physics departments. There really does seem to be a pattern in this unintentional process. I cannot help but think that an underlying model describes how we seek out and explore our creative interests, and that – as in nature – this behaviour is fractal.