# FISCAL POLICY AND DEBT MANAGEMENT WITH INCOMPLETE MARKETS


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# INTRODUCTION

- Government debt increased significantly in OECD countries in the aftermath of the 2008 crisis
- Economists and policymakers are concerned that these debt levels are ”too high”, debate what to do about it
- What is the ”right” level of debt?

# THIS PAPER

- Develop a theory of optimal public debt management
- Our focus:
  - what is the optimal/target debt level?
  - how quickly should gov't repay/accumulate its debt if it is above/below the target?
  - how much variability in gov't debt is optimal?
- Same questions for tax rates and tax revenues
  - follows from the budget constraint

# OUR FRAMEWORK

- Ramsey planner with distortionary taxation and incomplete markets
- Key insight: optimal debt level maximizes hedging possibilities offered by incomplete markets
- Derive explicit formulas (”sufficient statistics”) for
  - target debt level
  - speed of reversion to the target
  - variance of debt in ergodic distribution

# RESULTS

- Main formulas:

$$\text{target debt} = -\frac{\text{cov}\,(\text{returns, deficit})}{\text{var}\,(\text{returns})}$$

$$\text{speed of convergence} = \frac{1}{1+\beta^2\text{var}\,(\text{ returns})}$$

- Calibration to the US:
  - target debt level is negative but close to zero
  - speed of convergence slow (half life $\approx$ 500 years)
  - large variance of debt values in the invariant distribution
  - dynamics of debt and taxes in the data similar to the optimum

1. Complete markets: Lucas-Stokey (1983), ...
    - any debt level is optimal
2. Incomplete markets: Barro (1979), ...
    - any debt level is optimal (debt is random walk)
3. With sufficiently many assets can replicate complete markets: Angeletos (2004), Buera-Nicolini (2004)
    - see #1
4. Accumulate enough assets to never use taxes: Aiyagari et al (2002), Farhi (2010)
    - a knife-edge case
5. Nominal debt, possibilities of default
    - have not studied, but our insights should apply there too

# ENVIRONMENT

- Continuum of identical agents with preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ c_t - \frac{1}{1+\gamma} l_t^{1+\gamma} \right]$$

- No capital + exogenous gov't expenditures

$$c_t + g_t = l_t$$

- Gov't can use proportional tax $\tau_t$ and trade with agents one-period lived security at price $q_t$ with stochastic payoff $p_t$

$$g_t + p_t \mathsf{B}_{t-1} = \tau_t l_t + q_t \mathsf{B}_t$$

- i.i.d. shocks for $(g_t, p_t)$ , $\mathsf{B}_t$ is in a compact set
- Let $B_t \equiv q_t \mathsf{B}_t$, $R_t \equiv p_t / q_{t-1}$

## Lemma

*$\{c_t, l_t, R_t, B_t, \tau_t\}_{t=0}^{\infty}$ is a competitive equilibrium if and only if $\{l_t, B_t\}_{t=0}^{\infty}$ satisfies*

$$\underbrace{l_t - l_t^{1+\gamma}}_{=\tau_t l_t} + B_t = R_t B_{t-1} + g_t$$

- Easier to express hours as a function of tax revenues $Z$

$$\begin{aligned} Z &\equiv l(Z) - l(Z)^{1+\gamma} \\ \Psi(Z) &\equiv \frac{1}{1+\gamma} l(Z)^{1+\gamma} \end{aligned}$$

- Consumption is a residual

$$c_t = (1+\gamma)\Psi(Z_t) + R_t B_{t-1} - B_t$$

- Bellman equation (state $s = (g, p)$) :

$$V(B_-) = \max_{\{Z(s),B(s)\}} \mathbb{E}\left[RB_- - B + \gamma\Psi(Z) + \beta V(B)\right]$$

  subject to

$$Z(s) + B(s) = \underbrace{R(s)\,B_- + g(s)}_{\equiv E(B_-,s)} \text{ for all } s$$

- Policy functions $\tilde{B}(B_-, s)\,, \tilde{Z}(B_-, s)\,, \tilde{\tau}(B_-, s)$ induce optimum $\left\{\tilde{B}_t, \tilde{Z}_t, \tilde{\tau}_t\right\}_t$

**Monotonicity:** $\tilde{B}, \tilde{Z}, \tilde{\tau}$ are increasing in $E$

**Distortion smoothing:** $V'\left(\tilde{B}_t\right) = \mathbb{E}V'\left(\tilde{B}_{t+1}\right) + \beta\text{cov}\left(R_{t+1}, V'\left(\tilde{B}_{t+1}\right)\right)$

**Uniqueness:** $\tilde{B}_t$ converges to a unique invariant distribution

- Our goal: characterize properties of the invariant distribution

- Amount of risk depends on debt level:

$$E(B_-, s) = R(s) B_- + g(s)$$

- Let $B^*$ be the debt level that minimizes $\text{var}(E(B, \cdot))$ :

$$B^* = -\frac{\text{cov}(R, g)}{\text{var}(R)}$$

- Special case: possibility of perfect hedging, i.e. $R \in \mathcal{R}^*$ where

$$\mathcal{R}^* \equiv \{R : \text{there exists } B \text{ s.t. } E\left(B, \cdot\right) \text{ is constant}\}$$

- Monotonicity of policy rules:

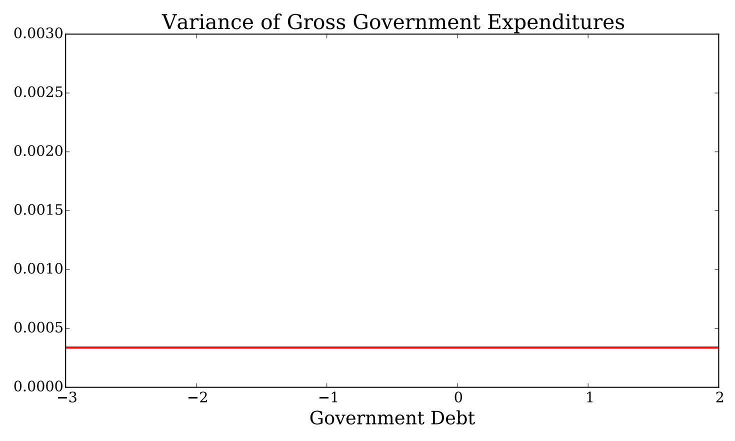
$$\begin{aligned} B &< B^* \Longrightarrow \text{cov}\left(R\left(\cdot\right), V'\left(\tilde{B}\left(B, \cdot\right)\right)\right) > 0 \\ B &= B^* \Longrightarrow \text{cov}\left(R\left(\cdot\right), V'\left(\tilde{B}\left(B, \cdot\right)\right)\right) = 0 \\ B &> B^* \Longrightarrow \text{cov}\left(R\left(\cdot\right), V'\left(\tilde{B}\left(B, \cdot\right)\right)\right) < 0 \end{aligned}$$

- Unique invariant distribution (follows from MCT)

$$\begin{aligned} \tilde{B}_t &\rightarrow B^* \\ \text{var}\left(\tilde{Z}_t\right) &\rightarrow 0 \\ \text{var}\left(\tilde{\tau}_t\right) &\rightarrow 0 \end{aligned}$$

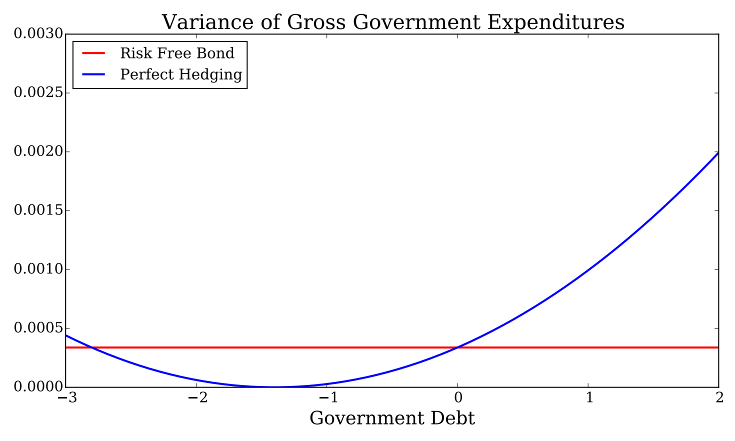
Risk Free Bond:

$$V'(B_t) = \mathbb{E}_t V'(B_{t+1})$$



Perfect Hedging:

$$V'(B_t) = \mathbb{E}_t V'(B_{t+1}) + \text{cov}_t(R_{t+1}, V'(B_{t+1}))$$

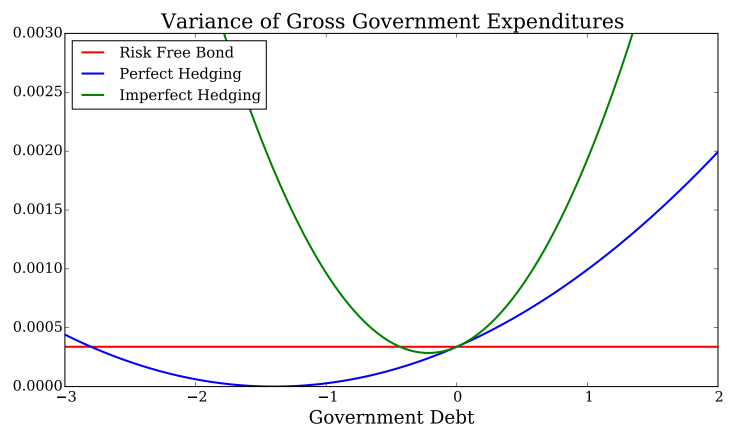


- When $R \notin \mathcal{R}^*$ a sequence of shocks can take debt away from hedging-maximizing level

- Our approach:
    - let $\check{B}(B, \cdot)$ be quadratic approximation of $\tilde{B}(B, \cdot)$ around $B$
    - study invariant distribution induced by $\check{B}$

Imperfect Hedging:

$$E\left(B_t, s\right) = R\left(s\right) B_t + g\left(s\right)$$



- The mean of the invariant distribution

$$\mathbb{E}\check{B}_t = B^*$$

- Speed of mean reversion

$$\mathbb{E}_0\left(\check{B}_t - B^*\right) = \left(\frac{1}{1+\beta^2 var(R)}\right)^t \left(\check{B}_0 - B^*\right)$$

- Variance of the invariant distribution

$$var\left(\check{B}_t\right) = \frac{var\left(E\left(B^*\right)\right)}{var(R)}\left(1+\beta^2 var(R)\right)$$

- The mean of the invariant distribution

$$\mathbb{E}\check{Z}_t \equiv Z^* = \mathbb{E}g + \frac{1-\beta}{\beta}B^*$$

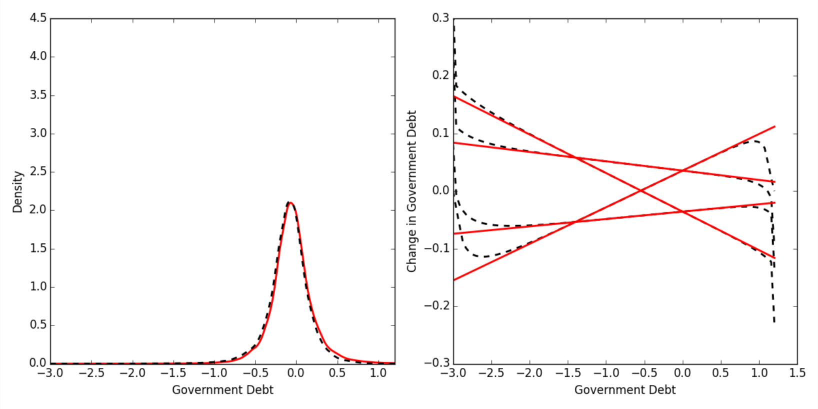
- Speed of mean reversion

$$\mathbb{E}_0\left(\check{Z}_t - Z^*\right) = \left(\frac{1}{1+\beta^2 var\left(R\right)}\right)^t \left(\check{Z}_{t-1} - Z^*\right)$$

- Variance of the invariant distribution

$$var\left(\check{Z}_t\right) = \left(\frac{1-\beta}{\beta}\right)^2 var\left(\check{B}_t\right)$$

# ACCURACY CHECK



- Target debt level: maximizes hedging
    - target level is positive if $cov(R, g) < 0$
    - target level is negative (accumulate assets) if $cov(R, g) > 0$

- Speed of mean reversion is determined by $var(R)$
    - $var(R) = 0$ implies debt is random walk as in Barro (1979)

- The less hedging $B^*$ offers, the bigger the variance of the invariant distribution

- For $\beta$ close to one, $var\left(\check{Z}_t\right)$ and $var(\check{\tau}_t)$ is close to 0 $\Longrightarrow$ all adjustment to shock is done via debt

- Equivalent expressions with iid shocks/quasi-linear preferences

$$\begin{aligned} B^* &= -\frac{cov\left(R,g\right)}{var\left(R\right)} = -\frac{cov\left(R_t, \mathbb{E}_t \sum_s^\infty \beta^s g_{t+s}\right)}{var\left(R_t\right)} \\ &= -\frac{cov\left(R_t, \mathbb{E}_t \sum_s^\infty \beta^s \left[g - Z^*\right]\right)}{var\left(R_t\right)} \end{aligned}$$

- More generally, the last formula applies

# EXTENSIONS

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- Persistent shocks
- Longer maturities
- Arbitrary market structure
- Redistribution
- Risk aversion

# EXTENSIONS

- Persistent shocks: same results
  - Hedge innovations in present value of government expediture

- Longer maturities: same results
  - Returns given by $R_t = \frac{p_t + q_t}{q_{t-1}}$, $q_t$ is price of asset.

- Suppose there are $K$ assets with arbitrary payoffs
- If portfolio weights are fixed: problem isomorphic to 1 security case
- If portfolio weights are chosen optimally each $t$: provide expressions for both the level and portfolio weights
  - the target portfolio is still the one that minimizes risk (i.e. $var(E)$)
- Additional insights
  - assets payoffs satisfy full spanning condition: replicate complete markets
  - otherwise: target portfolio maximizes hedging, but speed of convergence to it is slower than with 1 asset

# REDISTRIBUTION

- Simplest model of redistribution: a group of households with no income and hand-to-mouth
    - utility is $U(c)$, $U$ is strictly concave, satisfies Inada conditions
- Gov't can use lump sum transfers $T_t$ (the same for both groups)
- Gov't has Pareto weight $\omega > 0$ on the poor

## Bellman Equation

$$V(B_-) = \max \mathbb{E}[RB_- - B + \gamma\Psi(Z) + \qquad \beta V(B)]$$

subject to

$$Z(s) \qquad + B(s) = R(s)B_- + g(s) \text{ for all } s$$

## Bellman Equation

$$V(B_-) = \max \mathbb{E}[RB_- - B + \gamma\Psi(Z) + \omega U(T) + \beta V(B)]$$

subject to

$$Z(s) - T(s) + B(s) = R(s)B_- + g(s) \text{ for all } s$$

## Bellman Equation

$$V(B_-) = \max \mathbb{E}[RB_- - B + \underbrace{\gamma\Psi(Z) + \omega U(T)}_{\text{Cost of } Z-T} + \beta V(B)]$$

subject to

$$Z(s) - T(s) + B(s) = R(s)B_- + g(s) \quad \text{for all } s$$

- Main insights
  - properties of $\check{B}_t$ are as before
  - extends results to $\check{T}_t$
- Fluctuations both in deadweight losses and in inequality are costly:
  - minimize variability in both $Z$ and $T$

- Main insights
  - properties of $\check{B}_t$ are as before
  - extends results to $\check{T}_t$
- Fluctuations both in deadweight losses and in inequality are costly:
  - minimize variability in both $Z$ and $T$
- Stark contrast with AMSS, Farhi, Battaglini-Coate
  - they consider representative agent economy with $T_t \geq 0$
  - their prescription: accumulate a lot of assets, set $\tilde{\tau} = 0$, use fluctuations in transfers to smooth agg shock
  - their result survives only if marginal utility of the poor does not depend on $T$

- Same environment as before except utility function is

$$\frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$$

- Major complication: asset returns depend on policy

- New implementability constraint

$$U_{c,t}\mathsf{B}_t + U_{c,t}c_t + U_{l,t}l_t = \frac{p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}} U_{c,t-1}\mathsf{B}_{t-1}$$

- Define
  - effective debt: $X_t = U_{c,t}\mathsf{B}_t$
  - effective return: $\mathsf{R}_t = \frac{p_t U_{c,t}}{\beta \mathbb{E}_{t-1} p_t U_{c,t}}$
  - effective primary deficit: $\Phi_t = U_{c,t}\left[g_t - \tau_t l_t\right] = -U_{c,t}c_t - U_{l,t}l_t$
- All can be written as functions of $c_t$

- Bellman equation:

$$V(X_-) = \max_{\{c(s),X(s)\}} \mathbb{E}\left[U(c, c+g) + \beta V(X)\right]$$

subject to

$$X(s) = \mathsf{R}(c,s)\, X_- + \Phi(c,s) \ \text{ for all } s$$

- Distortion smoothing:

$$V'\left(\tilde{X}_t\right) = \mathbb{E}_t V'\left(\tilde{X}_{t+1}\right) + \beta cov_t\left(\mathsf{R}_{t+1}, V'\left(\tilde{X}_{t+1}\right)\right)$$

- For a given $\tau$ define $\mathsf{R}_\tau$, $\Phi_\tau$ and

$$X_\tau = \frac{\beta}{1-\beta}\mathbb{E}\Phi_\tau$$

- $\tau^*$ that maximizes hedging satisfies

$$\tau^* = \arg\min_\tau \operatorname{var}\left(R_\tau X_\tau + \Phi_\tau\right)$$

**Note**

$X^* = X_{\tau^*}$ that maximizes hedging satisfies

$$X^* = -\frac{\operatorname{cov}\left(\mathsf{R}_{\tau^*}, \Phi_{\tau^*}\right)}{\operatorname{var}\left(\mathsf{R}_{\tau^*}\right)}.$$

- Cost of raising revenues is proportional to $U_c \Longrightarrow$ convert variables to effective units
- Perfect hedging: $X^*, \tau^*$ are constant, but debt fluctuates to offset fluctuations in $U_c$
- Risk-free debt: $\text{cov}\,(p, \Phi) = 0$ implies $\text{cov}\,(\mathsf{R}, \Phi) > 0$
    - with state - uncontingent payoffs the target debt level is negative

- Apply our analysis to model calibrated to U.S. economy with:
    - risk aversion
    - persistent TFP shocks

- Evaluate accuracy of hedging predictions for:
    - ergodic mean
    - speed of convergence
    - ergodic variation

- Preferences

$$\ln c + \frac{1}{1+2} l^{1+2}$$

- 1 asset, return are matched to returns of the U.S. gov't portfolio

- 2 shocks process
    - TFP shocks with error term

$$\varepsilon_t = \rho_\theta \varepsilon_{t-1} + \sigma_\theta \epsilon_{\theta,t}$$

    - payoff vector

$$p_t = 1 + \chi \epsilon_t + \sigma_p \epsilon_{p,t}$$

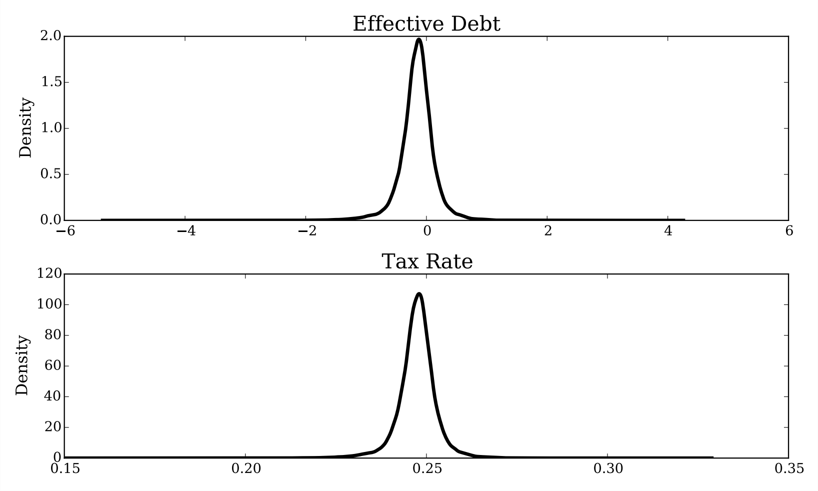
## Target Statistics:

- Dynamics of GDP
- Dynamics of returns to U.S. gov't portfolio

$$R_t = \frac{B_t + \text{primary surplus}_t}{B_{t-1}}$$

| Parameter | Value | Moment | Model | Data |
|---|---|---|---|---|
| | | Log Output | | |
| $\sigma_\theta$ | 0.14 | Std. Dev. | 0.015 | 0.015 |
| $\rho_\theta$ | 0.7 | Auto. Corr. | 0.56 | 0.57 |
| | | Returns | | |
| $\sigma_p$ | 0.048 | Std. Dev. | 0.035 | 0.035 |
| $\chi$ | 0.21 | Corr. with log(GDP) | -0.08 | -0.08 |
| $g$ | 0.25 | Gov. Spending/GDP | 0.25 | 0.25 |

# OPTIMAL POLICY: INVARIANT DISTRIBUTION



**Recall:**

$$\mathbb{E}_0\left(\check{x}_t - x^*\right) = \left(\frac{1}{1+\beta^2 var\left(\mathrm{R}\right)}\right)^t \left(\check{x}_0 - x^*\right)$$

Recall:

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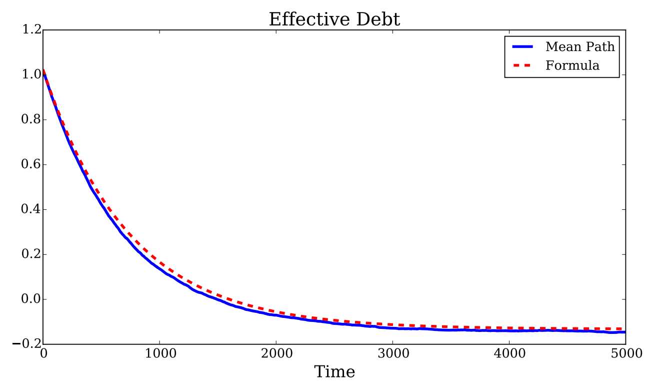

## Ergodic Distribution : Effective Debt ($x_t$)

| | Using Simulation | Using Formula |
|---|---|---|
| Mean | -0.148 | -0.133 |
| Half Life | 498 | 512 |
| Std. Deviation | 0.29 | 0.33 |

- Correlation of returns and output is close to 0:
  - correlation with effective returns is negative
  - accumulate assets
- Variability of effective returns is low and with larger orthogonal component
  - slow convergence to the mean
  - large variance of debt

# CONCLUSION

- Develop hedging theory of debt
  - Simple formulas for mean, variance and speed of convergence to ergodic distribution
- Predictions hold across range of environments
  - Multiple assets
  - Risk Aversion
  - Persistence
- Future Work:
  - Heterogeneous Agents (almost done)
  - Capital?

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- Rules:
  - Submit a paper each week (1 week prior)
  - Prepare a 5 minute presentation (no slides)
  - Fine to ask questions during presentation (encouraged)
  - Faculty can attend but won't talk (much)

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- Logistics:
  - Tuesday or Thursday 4pm
  - Name and email if you are interested