Macro Group

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Outline

- Related Literature
- The Model and Learning Specifications
- Estimation Technique of Public Signal
- Calibration, Estimation and Simulation Results
- Relationship to Public Signal usage and News Reporting
- Extensions

Intordution

- There has been evidence from past DSGE estimations that the introduction of learning and the relaxation of rational expectations can have a significant impact on parameter estimates and overall fit of the model compared to the data
- Milani (2005, 2007) shows that when rational expectations are exchanged for learning in a New-Keynesian model, the estimated parameters of indexation and other nominal frictions are close to zero. Suggesting that expectation formation modeling has significant effects in such models.
- Similar conclusions are found by Slobodyan and Wouters (2012a) who find that learning can fit business cycle fluctuations better when compared to rational expectations in more stylized DSGE models.

Intordution

- Two papers that play a key role in my procedures and findings are
  - Macroeconomic Expectations of Households and Professional Forecasters
    - Christopher Carroll (QJE, 2003)
  - Estimating a medium-scale DSGE model with expectations based on small forecasting models
    - Sergey Slobodyan and Raf Wouters (AEJ, 2012b)

Findings

- Tentative Title: The Effects of Information Dissemination on Macroeconomic Volatility
- Provides analytical estimates of the effects of professional forecasts on the volatility of Output, Inflation and other aggregate variables
- Further evidence that adaptive learning lowers the marginal likelihood in empirically estimated DSGE models.
  - Changes in parameter estimates and nominal frictions
- Correlation between when the public forecast signal is used and the vastness of news stories regarding economic forecasts of variables such as inflation, wage, investment, consumption and output growth

Expectations of Households and Professional Forecasters

- Carroll (2003) examines the formation of household inflation forecasts. Rather than assuming agents have complete understanding of the true economic model agents are assumed to obtain their inflation forecasts from the news media.

- Agents are assumed to absorb the economic news probabilistically so that economic news takes time to spread throughout the economy.

- Carroll's model provides a micro-foundation for Mankiw & Reis (2000) sticky information modeling assumption

Expectations of Households and Professional Forecasters

- Agents update their inflation expectations based on a Microfounded linear combination between their previous inflation expectations and the expectations of "professional" economic forecasters that are reported through the news media

$$M_t\left[\pi_{t,t+4}\right] = \lambda N_t\left[\pi_{t,t+4}\right] + (1-\lambda) M_{t-1}\left[\pi_{t-1,t+3}\right]$$

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- Carroll estimates the following regression as a proxy for the above equation using University of Michigan Inflation expectations and the SPF Inflation Forecast.

$$M_t\left[\pi_{t,t+4}\right] = \alpha_1 N_t\left[\pi_{t,t+4}\right] + \alpha_2 M_{t-1}\left[\pi_{t-1,t+3}\right]$$

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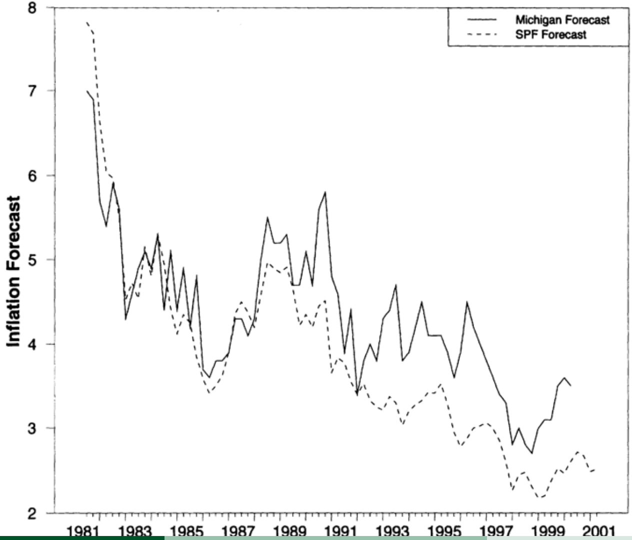
- Estimates $\alpha_1 + \alpha_2 = 1.02$ and with the restriction that $\alpha_1 + \alpha_2 = 1$ estimates a $\lambda = .73$ inline with the Mankiw & Reis estimate of sticky information

Expectations of Households and Professional Forecasters

- Carroll derives a news index of inflation and how often it is reported on using Nexis
- Finds that the squared gap between the UM inflation expectation and the SPF forecast is significantly bigger when the news index is lower
- Carroll Equation

$$GAP_t = \alpha_0 + \alpha_1 NEWS_t$$

Expectation Gap



Slobodyan and Wouters

- Empirically evaluates performance of a medium-scale DSGE model when agents form expectations by using small forecasting models

- $A \begin{bmatrix} Y_t \\ w_t \end{bmatrix} = B \begin{bmatrix} Y_{t-1} \\ w_{t-1} \end{bmatrix} + CE_t^* Y_{t+1} + D\varepsilon_t$

- Forecasting Forward Variable j:

$$y_{j,t} = b_{j,t-1} x_{j,t-1} + u_{j,t}$$

Slobodyan and Wouters

- Forecast Model i:
$$Y_t = \beta_{i,t-1} X_{i,t-1} + U_{i,t}$$

- In total five different small forecasting models are used

- The models β coefficients are aggregated to form expectations of variables as linear functions of a subset of Y_{t-1} variables

- $E_t^* Y_{t+1} = [\beta_{agg,t-1}]^2 X_{t-1}$

- $X_{t-1} = \Phi Y_{t-1}$

Slobodyan and Wouters

- DSGE Model Set Up

$$A \begin{bmatrix} Y_t \\ w_t \end{bmatrix} = B \begin{bmatrix} Y_{t-1} \\ w_{t-1} \end{bmatrix} + CE_t^* Y_{t+1} + D\varepsilon_t$$

- Actual Law of Motion (ALM)

$$A \begin{bmatrix} Y_t \\ w_t \end{bmatrix} = B \begin{bmatrix} Y_{t-1} \\ w_{t-1} \end{bmatrix} + C[\beta_{agg,t-1}]^2 \Phi Y_{t-1} + D\varepsilon_t$$

- DSGE Model Set Up

$$Y_t = \mu_t + GY_{t-1} + Hw_t$$

Five Forecasting Models

1. AR(1): every forward–looking variable is predicted based on its own lagged value;
2. AR(1) + 2: in addition to own lag, lagged interest rate and inflation are added to the RHS of every forecasting equation;
3. AR(2): every forward–looking variable is predicted based on two own lags;
4. AR(1) + 1: in addition to own lag, inflation is added to the RHS of every forecasting equation;
5. AR(1) + 3: in addition to own lag, interest rate, inflation, and output are added to the RHS of every forecasting equation.

BIC Weight Selection

- For every forecast model i agents track its performance

$$B_{i,t} = t \cdot ln\ det\left(\frac{1}{t}\sum_{\tau=1}^{t} U_{i,\tau}U'_{i,\tau}\right) + \kappa_i \cdot ln(t)$$

- Given $B_{i,t}$ the weight of a forecasting model is proportional to $exp(-.5 \cdot B_{i,t})$

- These weights are used to form $\beta_{agg,t-1}$

Model Solving Steps

Calculate beta coef. for each model under each data information set

Using time t PLM, solve for reduced form ALM:

$$Y_t = \mu_t + G_t Y_{t-1} + Hw_t$$

Using BIC weight criteria to calculate PLM

If G_t is explosive, ignore coef. update and use G_{t-1} and μ_{t-1} matrixes

Model Comparison in terms of Marginal Likelihood

REE model (TFP based output gap)	-926
KF Learning with 5 small models: same sample for beliefs and model estimation	
5 models, BIC selection, γ and σ estimated, $\rho = 1$	-917
5 models, EW combination, γ and σ estimated, $\rho = 1$	-910
5 models, BIC selection, γ and σ fixed, ρ estimated	-911
5 models, EW combination, γ and σ fixed, ρ estimated	-909
KF Learning with 5 small models: longer sample for beliefs than for model estimation	
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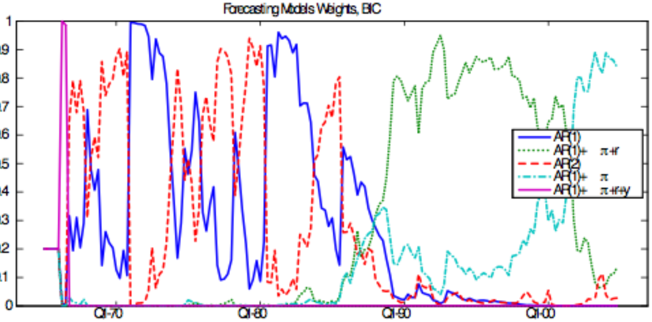
Forecasting Model weights

Figure 5: Weights of the individual model under the BIC selection criterium



The SWFF Model

- The estimated model contains 8 shocks and additional frictions including
  - Sticky price and wage setting that allow for inflation indexation
  - Habit formation in consumption
  - Investment adjustment costs and capital utilization
  - Financial frictions (along the lines of Bernanke, Gertler, Gilcrhist (1999) Financial Accelerator)
- The shocks in the model include: productivity shock, investment shock, price and wage mark up shocks, government spending, monetary policy shock, preference shock
- Finally, there is an Financial Spread Shock

Introducing Adaptive Learning

- Introduce learning into the SWFF Model. I assume that agents do not have perfect knowledge of the reduced form parameters, exogenous processes or steady state values of the model when forming expectations about the future
- Agents must form expectations about the path of 5 forward variables
  - Inflation
  - Consumption
  - Wages
  - Investment
  - Relative Price of Capital (Q)

Introducing Adaptive Learning

- Agents believe the economy follows one of the following laws of motions.

$$y_t = a_{1,t} + b_{1,t} y_{t-1} + e_{1,t}$$
$$y_t = a_{2,t} + c_{2,t} Y^*_{t|t-1} + e_{2,t}$$
$$y_t = a_{3,t} + b_{3,t} y_{t-1} + c_{3,t} Y^*_{t|t-1} + e_{3,t}$$

- The vector y_t contains the five forward looking variables in the model. The matrices $b_{1,t}$ and $b_{3,t}$ are square matrices whose off diagonal elements are equal to zero.

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$$

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- Equations 2 and 3 are where I introduce professional public perception of future variables into the DSGE model with the inclusion of Y^*.

- All non-zero coefficients in the 3 equations are calculated using constant gain learning and ROLS.

Introducing Adaptive Learning

- Agents are uncertain about weather or not to use the public forecast announcement.
- Thus agents use Bayesian weights to calculate the aggregate PLM. These Bayesian weights are derived by previous realizations of each models residuals.

$$B_{i,t} = t \cdot ln\ det\left(\frac{1}{t}\sum_{\tau=1}^{t} E_{i,\tau}E'_{i,\tau}\right) + \kappa_i \cdot ln(t)$$

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- If a model has produced large residuals over the recent past observations it will receive a lesser weight used in averaging across all PLMs.
    - I use a rolling window of residuals of 12 quarters

Introducing Adaptive Learning

- The inclusion of Bayesian weighting allows agents to choose between and weigh private signals derived from AR(1) processes (1), completely using the public signal (2) and using both the public signal and the private signal (3) when selecting their aggregate PLM.

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- The inclusion of Bayesian weighting allows agents to choose between and weigh private signals derived from AR(1) processes (1), completely using the public signal (2) and using both the public signal and the private signal (3) when selecting their aggregate PLM.

- The appropriately scaled public signal value will be added to the constant term in the actual law of motion (ALM)

$$\mu_t = a_{agg,t} + c_{agg,t} Y^*_{t|t-1}$$

Expectation Formation

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3. Agents use all $t-1$ information and the public forecasts previously announced to update the coefficients on each of their PLMs using constant gain ROLS.

Expectation Formation

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6. Time t exogenous shocks occur and all time t endogenous variables are then realized in the economy.

Estimation Technique of Public Signal Y^*

- Stock and Watson (2008) estimated a DFM using over 100 macroeconomic and financial time series across 7 latent factors.
- I use a similar data set to forecast the future values of the 5 forward endogenous variables in the model
- Some of these series include Leading Economic Indicators:
  - Bond Yields on AAA Corporations
  - US Treasury at different lengths
  - Housing Starts
  - Stock Price Indexes
  - Employment by Sector
  - Multiple Price and Production Indexs

Dynamic Factor Models

- Dynamic Factor Models (DFM) and Factor augmented VAR's have been shown to be very good at explaining variation in time series and short-term forecasting

$$X_t = \Lambda F_t + e_t$$

$$F_t = \psi F_{t-1} + \epsilon_t \text{ where } \epsilon_t \sim NID(0, I_m)$$

- F_t is a vector of estimated latent Factors, and X_t is the large collection of data series

- F_t, Λ and ψ are estimated using a Gibbs Sampler

Selecting the Number of Latent Factors

- Step 1: Bai and Ng (2002) Find the r tha minimizes the following IC

$$log\left(\frac{1}{NT}\right)(X - F_0 n\Lambda)(X - F_0\Lambda)' + r\frac{T+N}{TN} log\left(\frac{TN}{T+N}\right)$$

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- Step 2: Order the Eigenvalues of X'X from largest to smallest

$$BP = \left[\left(\tfrac{a}{N}\right) + \left(\tfrac{a}{T}\right)\right]^{-.75}\left(\frac{sum(1 : r - k)}{sum(:)}\right)$$

- smallest $k < r$ where $BP < 1$ (Breitburg et al. (2010))

Estimation Technique of Public Signal Y^*

- The vector Y^* is announced in the economy to the agents and provides expected values of the forward variables derived by a DFM
- In the context of the model these datasets can be thought of as holding noisy information about the past and future values of the unobserved structural shocks in the model.
- Once the economy enters a new period the previous periods observations are included in the large dataset and the dynamic factor model (DFM) is re-estimated and new public forecasts are announced for the appropriate variables.

Estimation Technique of Public Signal Y^*

- **Why Use a DFM to estimate the public signal?**

Estimation Technique of Public Signal Y^*

- **Why Use a DFM to estimate the public signal?**
- I have found that the forecasts produced by the DFM specification are correlated with the Greenbook forecasts of professional forecasters (SPF) for inflation and quarter-to-quarter growth of Output.
- Stock and Watson (2012) have shown that a Bayesian estimated DFM outforecasts many other models over the shot horizon (3-6 months)

Estimation Technique of Public Signal Y^*

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- **Why not just use the SPF forecasts instead?**

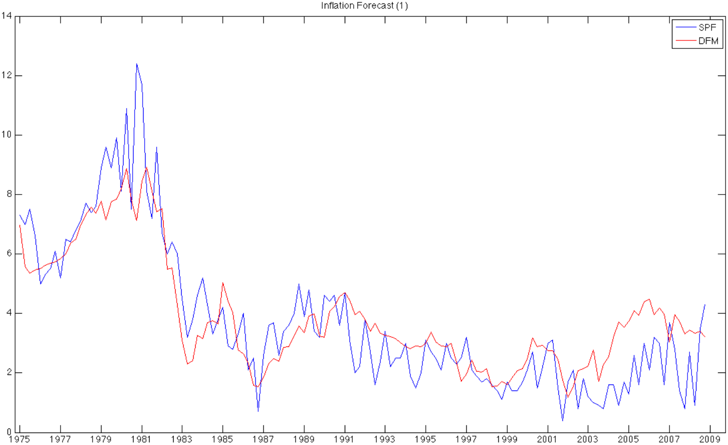
Estimation Technique of Public Signal Y^*

- **Why Use a DFM to estimate the public signal?**
- **Why not just use the SPF forecasts instead?**
- A DFM allows me to proxy for the SPF forecasts, a process which I can continue to use in simulations of the future.

SPF and DFM Forecast Correlation

Series	1 Quarter Forecast	2 quarter Forecast
Inflation	0.88	0.86
Output Growth	0.28	0.57
Consumption Growth	0.03	0.33
Unemployment Rate	0.90	0.88

SPF and DFM Forecasts



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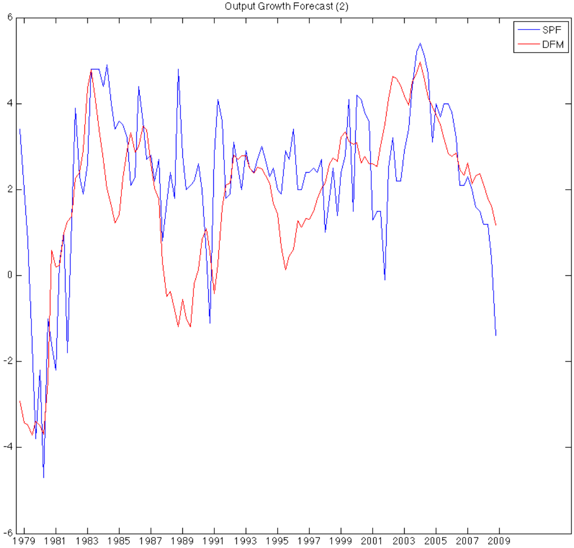


Calibrations

- Different ways to initialize agents initial beliefs $\beta_{1|0}$ and $R_{1|0}$
  - Can assume agents start out at REE
  - Can use a training sample (pre-estimation period) to initialize beliefs (1975-1984)

Calibrations

- Different ways to initialize agents initial beliefs $\beta_{1|0}$ and $R_{1|0}$
  - Can assume agents start out at REE
  - Can use a training sample (pre-estimation period) to initialize beliefs (1975-1984)
  - A projection facility is used to assume the eigenvalues of G are all less than one
    - If G is non-stationary agents ignore the updated β coefficients and refer back the previous periods coefficients tp calculate G
    - The projection facility does not play a big role in the estimation window (1984-2011) but does come into play about 10% of the time in the Simulations

Estimation

- The SWFF model is estimated under rational expectations and different adaptive learning rules
- I estimate the model using a Metropolist Hastings sampler with 250000 draws including burn-in period of 50000 draws
- Standard priors are used for the structural parameters and the model is fit to 8 data sets over the period of 1984Q2-20011

Bayesian Model Selection

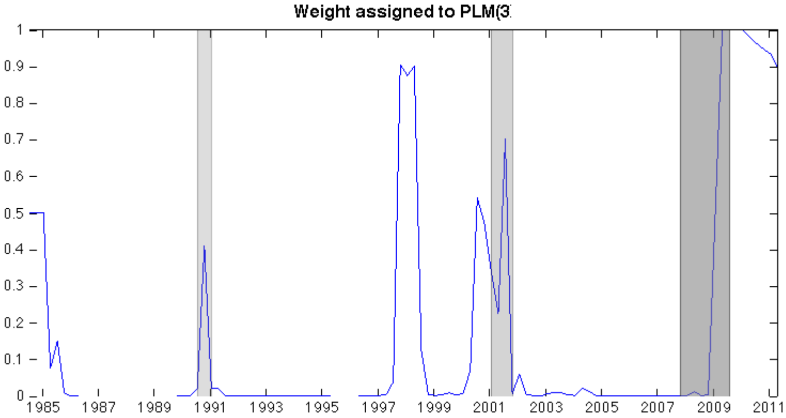
	PLM(1)	BW	PLM(3)	EW	REE
Marginal likelihood	-776.75	-778.54	-775.93	-775.74	-846.69
NW Standard Error	0.265	0.209	0.038	0.497	0.086
Model Probability	0.1617	0.027	0.3672	0.444	$1.1x10^{-31}$

- Marginal likelihood calculated using the Modified Harmonic Mean estimator

Parameter Estimates

Parameter	Description	REE	BW
γ_p	Degree of price indexation	.17	.05
ξ_p	Calvo price stickiness	.9	.85
ξ_p	Calvo wage stickiness	.93	.74
h	Habit consumption	.85	.71
ρ_b	AR(1) Consumption coef.	.74	.58
ρ_b	AR(1) Investment coef.	.73	.51
σ_b	Std of Con. shocks	.09	.53
σ_I	Std of Inv. shocks	.895	2.89
σ_p	Std of Price shocks	.08	.193
σ_w	Std of Wage shocks	.03	.192

Weight assigned to PLM(3)



Simulation Procedures

- I start every simulation at 2011Q3 and simulate for 450 quarters into the future and use the posterior mode estimates for each parameter

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- I start every simulation at 2011Q3 and simulate for 450 quarters into the future and use the posterior mode estimates for each parameter
- After each iteration of the DSGE model I take the 8 observable variables generated in the DSGE model that are also in the dataset of the DFM and randomly pick k of them to solve for k factors of time t (f_t).

$$\begin{bmatrix} Output\,(X_1) \\ Inflation\,(X_2) \\ Consumption\,(X_3) \\ Investment\,(X_4) \\ ------ \\ Housing\ \ Starts \\ Labor\ \ Productivity \\ Unemployment\ \ Rate \end{bmatrix} = \Lambda \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ ---- \\ e_{96} \\ e_{97} \\ e_{98} \end{bmatrix}$$

Simulation Procedures

- These time t factors are then used with the estimated factor loading matrix (Λ) to solve for the remaining time t observations of the DFM dataset (X_t) that are not determined inside the DSGE model.
- Due to computational time the DFM parameters (Λ, F, ψ) are only re-estimated once a year

Simulation Procedures

- The zero lower bound is protected using shadow monetary policy shocks using an algorithm outlined by Holden(2013)

$$Y_t = \mu_t + GY_{t-1} + H \left[\begin{bmatrix} v_a \\ v_b \\ v_g \\ v_F \\ v_I \\ v_w \\ v_p \\ v_R \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \alpha V_{zlb} \end{bmatrix} \right]_t$$

- If the v_t shocks do not result in a negative R_t than $\alpha = 0$
- If the v_t shocks do result in a negative R_t than $V_{zlb} = 1$ and the appropriate value of α is found to assure $R_t = 0$

Simulation Procedures

- Four different expectation procedures are simulated
  - PLM(1)
  - Bayesian Weights on each PLM
  - Bayesian Weights with a small noise shock around the public signal
  - Bayesian Weights with a large noise shock around the public signal

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- Four different expectation procedures are simulated
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- $Y_t = a_{3,t} + b_{3,t} Y_{t-1} + c_{3,t} \left(Y^*_{t|t-1} + \eta_t \right) + e_{3_t}$

- Where $\eta_t \sim NID(0, \sigma_\eta)$

Simulation Results (Standard Deviations)

Series	PLM(1)	BW	BW (low noise)	BW (High noise)
Inflation	0.446	0.403	0.433	0.471
Output Growth	1.05	0.99	1.04	1.14
Consumption Growth	0.63	0.625	0.63	0.65
Interest Rate	0.89	0.84	0.88	0.93
Investment Growth	3.71	3.58	3.66	3.87
Wage growth	0.27	0.268	0.27	0.274

Future Extensions

- Great Question