

**Macro Group**  
**Adam Check**  
**October 24, 2014**

# Plan for Talk

- Quantity Theory of Money & Breakdown
- "Two Illustrations of the Quantity Theory of Money: Breakdowns and Revivals"
  - Sargent & Surico (2011)
- Preliminary Idea

# Quantity Theory of Money

- All else equal, an increase in the money supply leads to a one-for-one increase in the inflation rate.
- Formally:

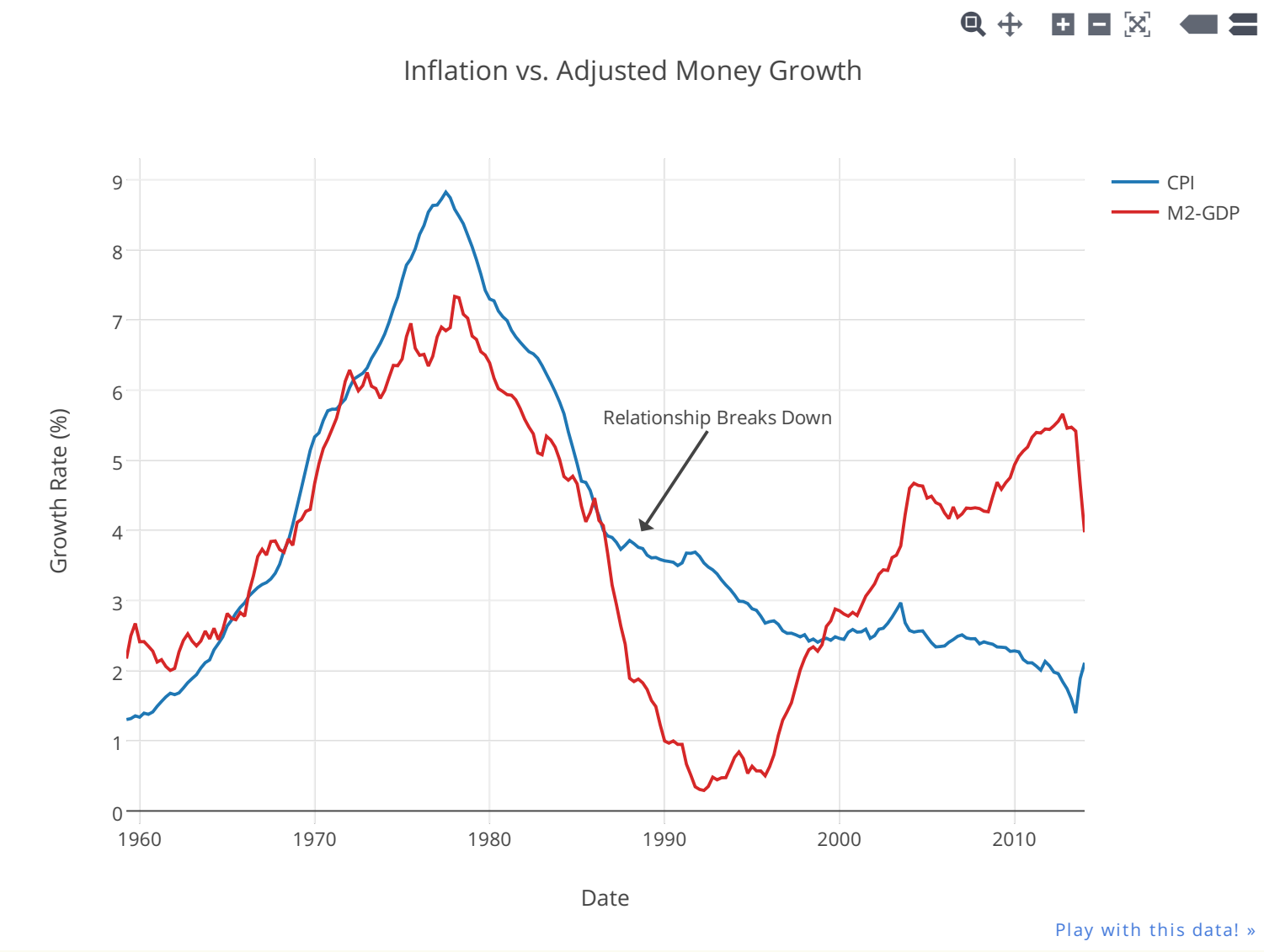
$$MV = PY$$

$$\% \Delta M + \% \Delta V = \% \Delta P + \% \Delta Y$$

$$\% \Delta P = \% \Delta M - \% \Delta Y$$

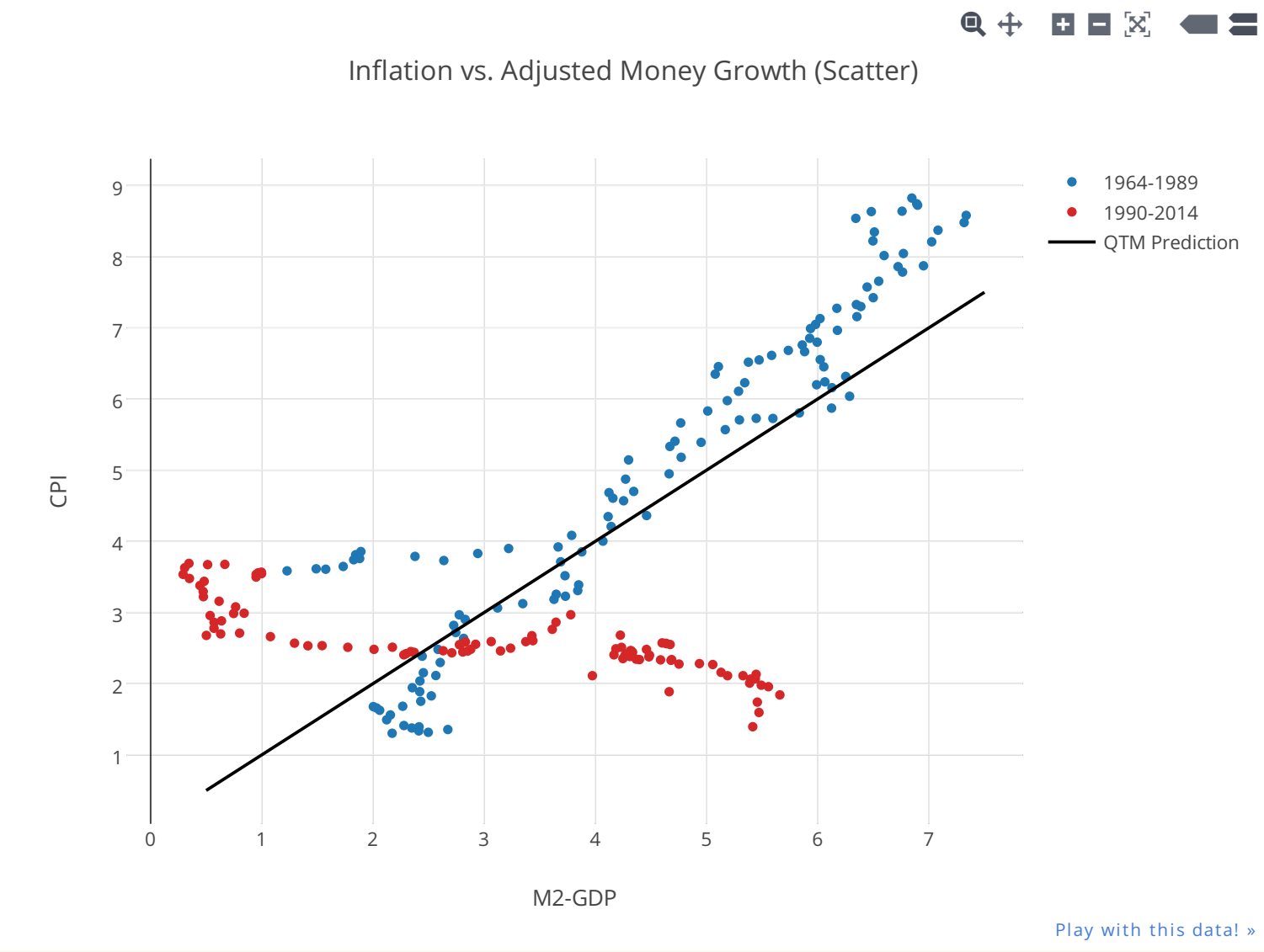
- One of the most well known relationships in all of macroeconomics.

# Quantity Theory of Money



[Play with this data! »](#)

# Quantity Theory of Money



# Quantity Theory of Money

## Breakdown not only in U.S.

- Teles and Uhlig (2011) find that this breakdown happens in virtually every advanced economy with low trend inflation.
  - True even after accounting for interest rates and money demand.

## What accounts for the breakdown?

- Two theories:
  1. Changes in Central Bank Policy
  2. Mismeasurement of Money
    - Ireland (2014), Lucas and Nicolini (2012)

# Sargent & Surico (2011)

- More formally show that the correlation between money and inflation changes significantly over time.
- Use a small DSGE model and show that central bank policy can drive the correlation between money and inflation down towards zero.

# Moving Average

- First, calculate the moving average of a series using the method of Lucas(1980):

$$\bar{x}_t(\beta) = \alpha \sum_{k=-n}^n \beta^{|k|} x_{t+k}$$

$$\beta \in [0, 1)$$

$$\alpha = \frac{(1 - \beta^2)}{1 - \beta^2 - 2\beta^{n+1}(1 - \beta)}$$



# A Digression on Spectral Densities

- Next, consider a regression of some variable  $y_t$  on the infinite past and future lags of another variable  $z_t$ :

$$y_t = \sum_{j=-\infty}^{\infty} h_j z_{t-j} + \epsilon_t$$

where  $\epsilon_t$  satisfies the usual orthogonality conditions.

# A Digression on Spectral Densities

- We can convert this time series regression to a spectral density regression with the Fourier transform (ch. 6 in Hamilton):

$$\tilde{h}(\omega) = \sum_{j=-\infty}^{\infty} h_j \exp(-i\omega j)$$

and we have:

$$\tilde{h}(\omega) = \frac{S_{yz}(\omega)}{S_z(\omega)}$$

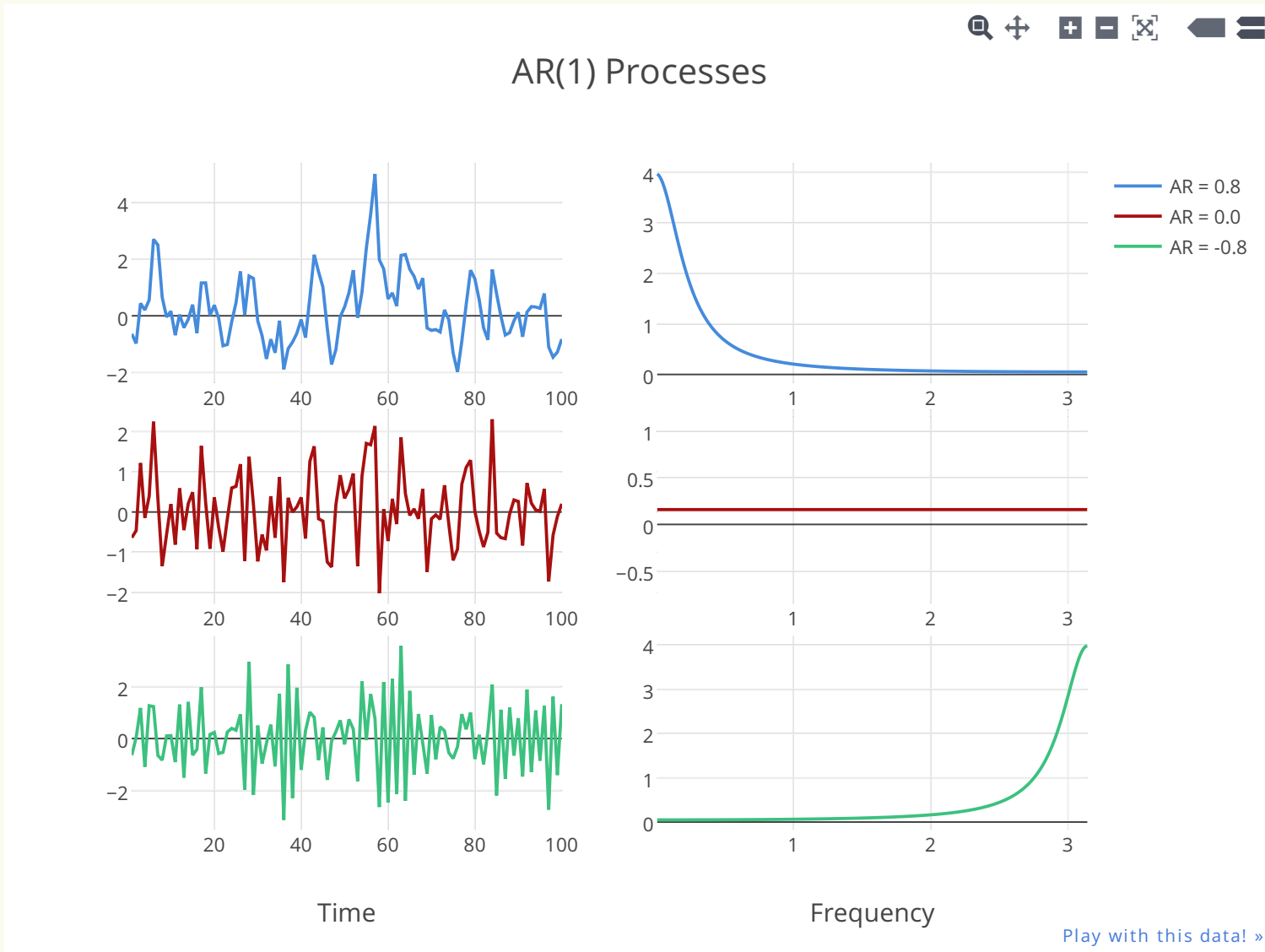
where  $S_z(\omega)$  is the spectral density of  $z$  evaluated at  $\omega$ , and  $S_{yz}(\omega)$  is the cross-spectral density of  $y$  and  $z$  evaluated at  $\omega$ .

# A Digression on Spectral Densities

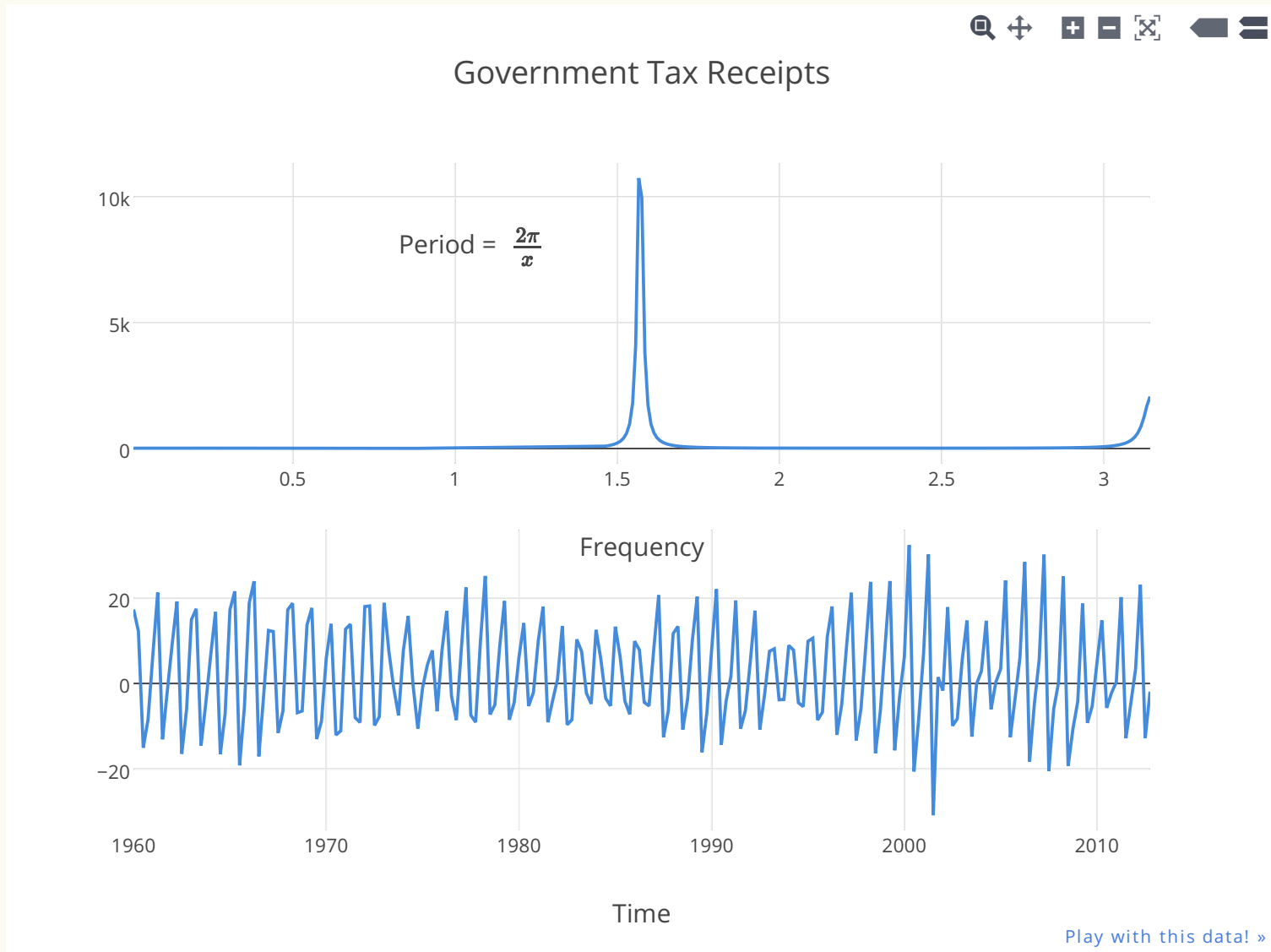
- Finally, the sum of the distributed lag regression coefficients is given as:

$$\sum_{j=-\infty}^{\infty} h_j = \tilde{h}(0) = \frac{S_{yz}(0)}{S_z(0)}$$

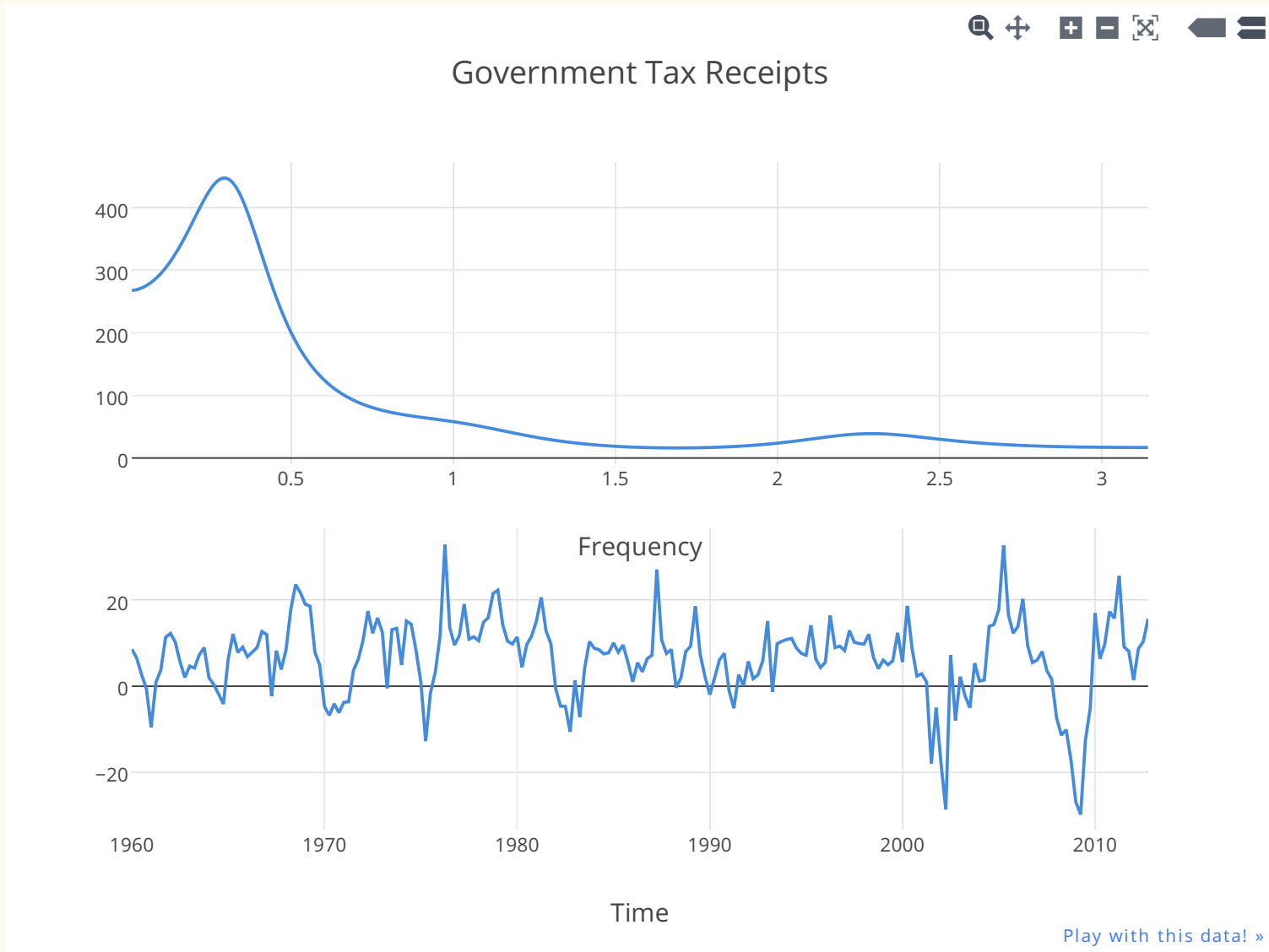
# What Does a Spectral Density Look Like?



# What Does a Spectral Density Look Like?



# What Does a Spectral Density Look Like?



# Back to the Lucas Moving Average:

- Whiteman (1984) showed that for  $\beta$  near 1, the regression coefficient,  $b_f$ , of one Lucas moving average,  $\bar{y}_t(\beta)$  on another,  $\bar{x}_t(\beta)$ , satisfies:

$$b_f \approx \frac{S_{yx}(0)}{S_x(0)} = \tilde{h}(0)$$

# Why do so much work?

- Can directly map the coefficients from VAR and DSGE models into  $\tilde{h}(0)$ .
  - Once the DSGE is solved, do not have to simulate the model in the future to find long-run correlations.
  - Instead, simply map the parameters into  $\tilde{h}(0)$  to find the approximate long-run correlations between the variables of interest.
  - Saves computational time and reduces subjectivity.



# Mapping

- Given the multivariate linear system of equations:

$$X_{t+1} = AX_t + BW_{t+1}$$

$$Y_{t+1} = CX_t + DW_{t+1}$$

- Then we have the mapping:

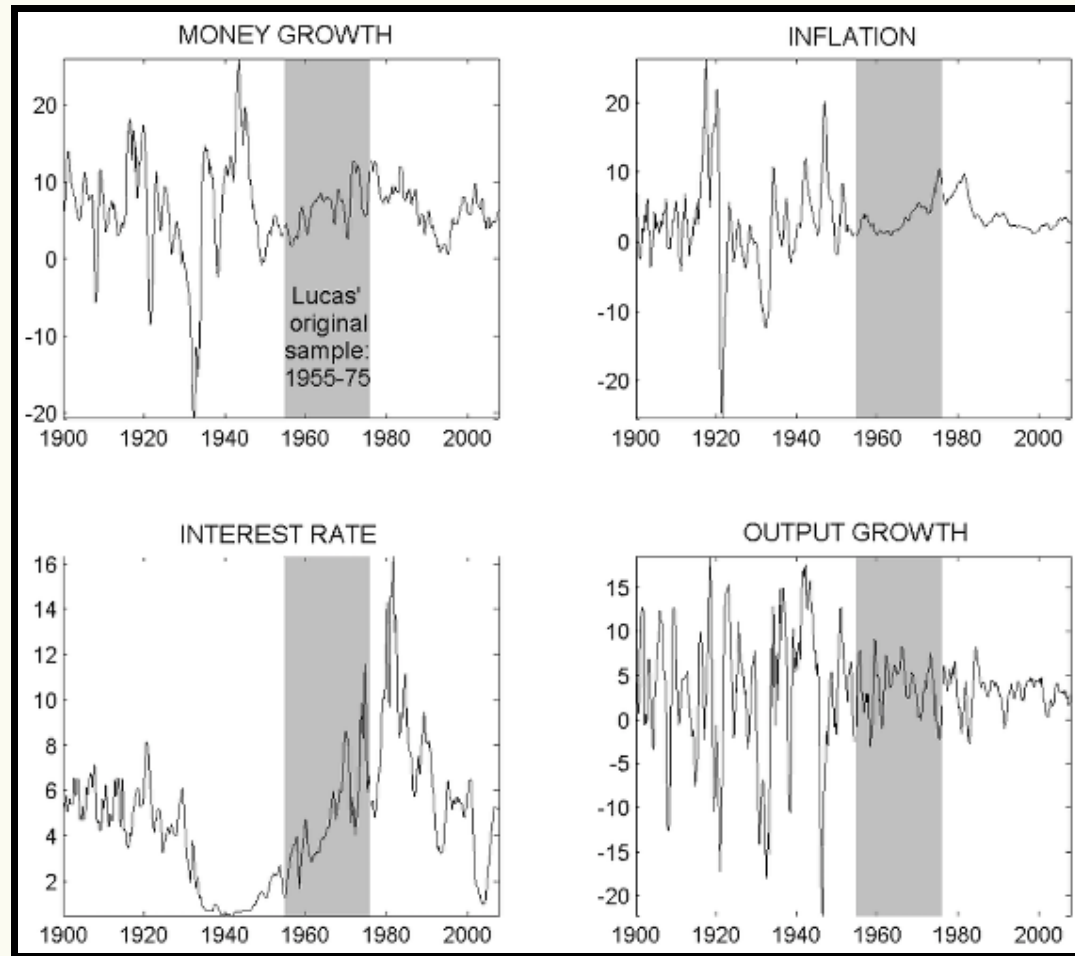
$$S_Y(\omega) = C(I - A \exp(-i\omega))^{-1} BB' (I - A'^{i\omega})^{-1} C' + DD'$$

To find the spectral densities of interest,  $S_y$ ,  $S_x$ , and  $S_{yx}$ , we simply pick out the relevant elements of the  $S_Y$  matrix.

# Data

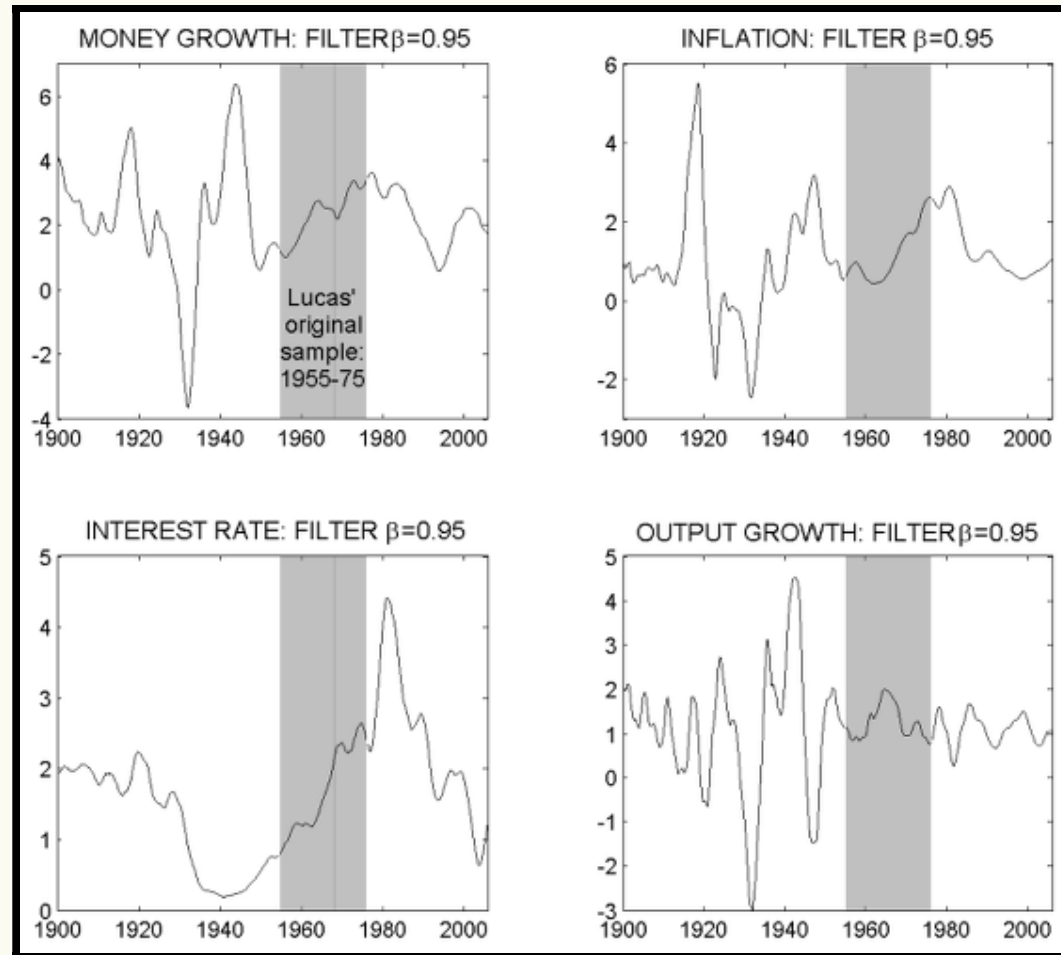
- Now that we know how to recover the long-run regression coefficients, let's take a look at the data.
- M2 growth, GDP/GNP deflator, 6 month interest rate on commercial paper, and output growth.
  - From 1900-2005

# Data



Variables Over Time

# Data

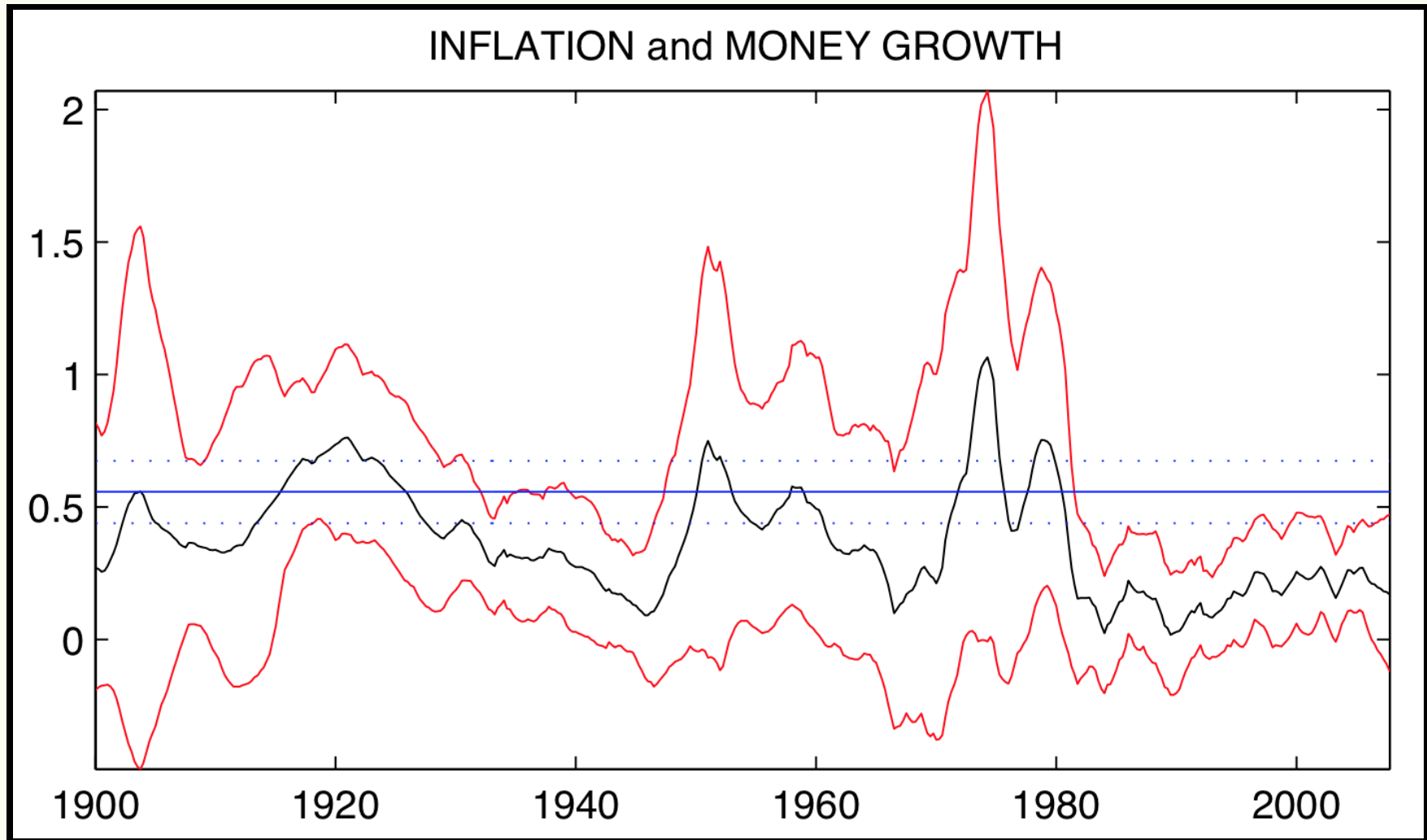


Filtered Variables Over Time

# Measuring the Correlations

- Remember, we are primarily interested in the long run correlation between money and inflation (they also investigate the correlation between money and interest rates).
  - Sargent and Surico estimate a time varying parameters VAR (TVP-VAR) with stochastic volatility using Bayesian methods, a-la Primiceri (2005)
  - Could give a whole presentation on that methodology.
  - This allows them to compute an estimate of  $\tilde{h}(0)$ , the approximation to the long-run correlation, at each point in time.

# Measuring the Correlations



Inflation and Money Growth

# Measuring the Correlations

- Clearly, the correlation between money and inflation is changing over time.
- But why?
  - Sargent and Surico suggest that it could be due to changes in monetary policy.
  - Estimate and perform policy experiments in a small scale DSGE model. Is it possible for changes in Fed policy to impact these correlations? If so, how?

# Log-Linearized DSGE

## Three Main Structural Equations:

$$\pi_t = \theta(1 - \alpha_\pi)E_t\pi_{t+1} + \theta\alpha_\pi\pi_{t-1} + \kappa x_t - \frac{1}{\tau} e_t$$

$$x_t = (1 - \alpha_x)E_t x_{t+1} + \alpha_x x_{t-1} - \sigma(R_t - E_t\pi_{t+1}) + \sigma(1 - \xi)(1 - \rho_a)a_t$$

$$\Delta m_t = \pi_t + z_t + \frac{1}{\sigma\gamma} \Delta x_t - \frac{1}{\gamma} \Delta R_t + \frac{1}{\gamma} (\Delta\chi_t - \Delta a_t)$$

where the third equation is a money demand equation.



# Log-Linearized DSGE

Auxiliary Equations:

$$\tilde{y}_t = x_t + \xi a_t$$

$$\Delta y_t = \tilde{y}_t - \tilde{y}_{t-1} + z_t$$

# Log-Linearized DSGE

## Shocks:

$$e_t = \rho_e e_{t-1} + \varepsilon_{et}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_{at}$$

$$\chi_t = \rho_\xi \xi_{t-1} + \varepsilon_{\chi t}$$

$$\Delta \ln Z_t \equiv z_t = \varepsilon_{zt}$$

where all  $\varepsilon_{kt}$  are distributed i.i.d.  $N(0, \sigma_k)$ , each having its own variance.

# Log-Linearized DSGE

## Closing the Model with a Policy Rule:

- Sargent & Surico use one of two possible monetary policy rules to close the model.
- They use either a money supply targeting rule:
$$\Delta m_t = \rho_m \Delta m_{t-1} + (1 - \rho_m)(\phi_\pi \pi_t + \phi_x x_t) + \varepsilon_{mt}$$
- Or the more standard interest rate targeting Taylor rule:
$$R_t = \rho_r R_{t-1} + (1 - \rho_r)(\psi_\pi \pi_t + \psi_x x_t) + \varepsilon_{Rt}$$

# Estimation Procedure

- Performed Bayesian estimation of the model a la An & Shorfeide (2007)
- Estimated over the period 1960-1983Q4
  - This is a time period over which the quantity theory still held.
    - Will be important for policy counterfactuals.
  - Estimated using the money supply targeting rule, instead of the Taylor rule.

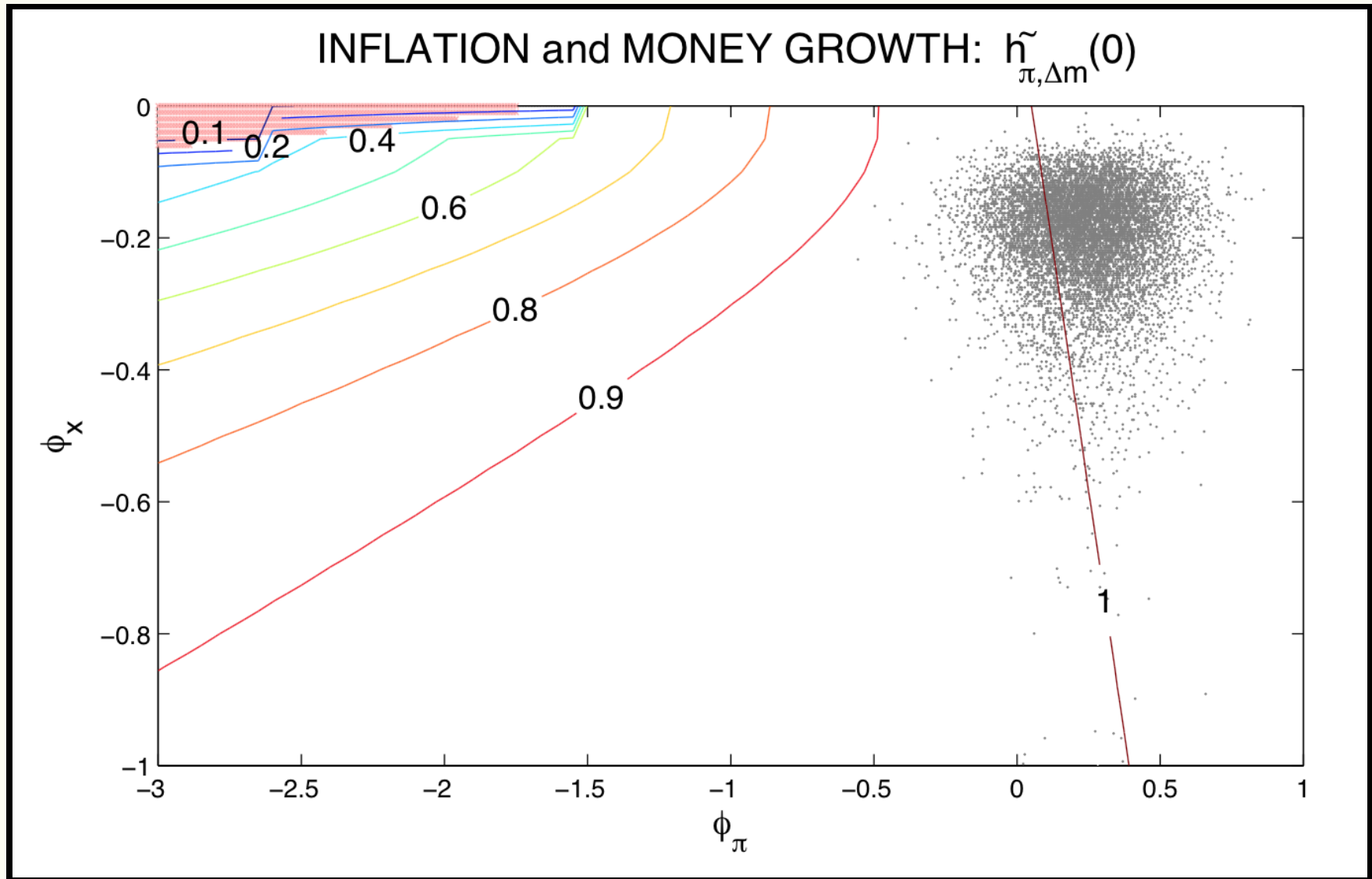
# Policy Experiment

- Hold all estimated parameters at their posterior mean values, except for the two parameters in the monetary policy rule.
- Numerically compute the solution to the model over a 2-D grid of policy parameters  $(\phi_\pi, \phi_x)$ .
  - Vary the response to inflation over  $[-3.0, 1.0]$
  - Vary the response to output over  $[-1.0, 0.0]$
- After computing the solution, compute the value of the spectral density regression to approximate the low frequency correlation coefficient between inflation and money growth,  $\tilde{h}(0)$ .

# Results

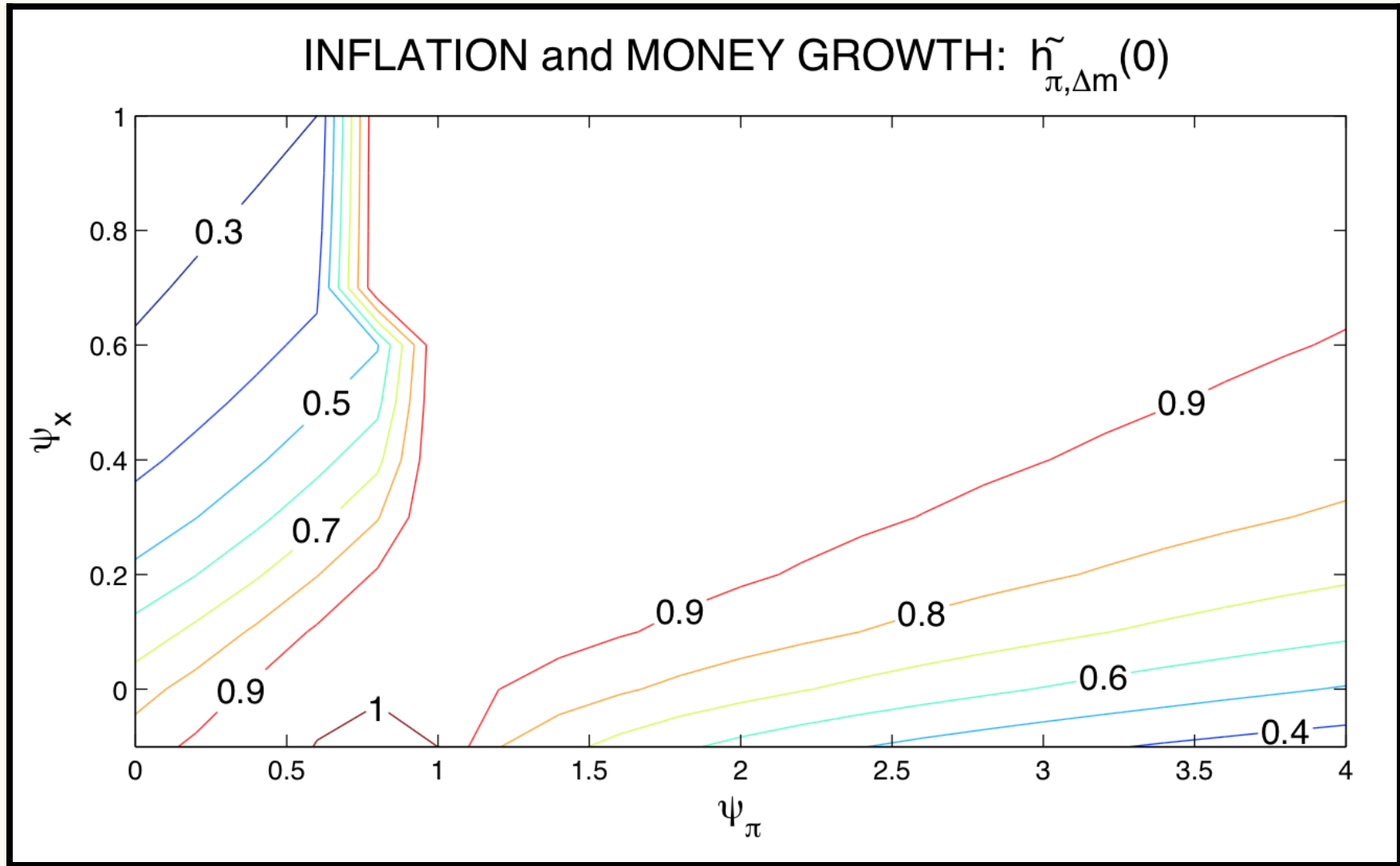
- Presented as a contour plot.

# Results



Gray dots represent draws from the posterior estimation over

# Results



Correlation & Policy in a Taylor rule



# Results

- Sargent & Surico also show results for the case when they reduce the variance of the supply shock (since most estimates suggest that the variance of the supply shock fell after 1983 during the great moderation).
- Similar results, but the same combination of inflation, output coefficients produces a correlation about 0.1 higher than when the variance of that shock is high.

# Conclusion

*"As citizens, we prefer times when the propositions have broken down because Lucas's unit slopes can be expected to emerge when a monetary authority...respond[s] too weakly to inflationary pressures."*

- In theory, a change in central bank policy can account for a change in the long-run correlation between money growth and inflation.

# Weaknesses

- Show that policy from '60-'83 was consistent with the QTM.
- Do not show the counterfactual, that policy from '84-'05 was consistent with a low correlation.
- Result strongest for money supply rule, which is nonstandard.
- Only change analyzed was a shift in the strength of the response.

# My Idea

- Attempt to more satisfactorily answer the questions:
  1. Exactly how has Fed policy changed over 1960-2014?
  2. How much of the reduction in the quantity theory correlation can the shift in coefficients around the late 1980s/early 1990s explain?

# Estimating Fed Policy

- Use recently developed "Time Varying Dimension" (Chan et al. 2012) or "Dynamic Model Averaging" (Koop & Korobilis 2012) techniques to estimate a time-varying Taylor rule more accurately than has been done before.
  - TVP is overparameterized (for Taylor rule, even constant parameters is...)
  - TVD allows for Bayesian shrinkage of coefficients towards zero.
    - In practice, yields models with a lower MSFE.
- In addition to strength of response, also allow for forward and backward looking policy rules.

# Explaining the Reduction

- Use the same (or similar) model as Sargent and Surico.
  - Richer policy experiments.
    - Solve model for different combinations of forward/backward looking rules.
  - Combined with estimated parameters, see how much of the reduction a change in Fed policy can explain.

# Preliminary Work

- Has the Fed shifted to be more forward looking post 1990?
  - Performed Bayesian estimation of a Stochastic Search Variable Selection model (George, Sun, and Ni 2008)
  - Imposed a break date of 1992 (based on an inner-ocular test as suggested by Wilson (2014)).
- If the Fed shifted from backward to forward looking, how would that affect the correlation in the DSGE model estimated by Sargent & Surico?

# Preliminary Results

- Fed much more likely to include inflation forecasts in its policy rule after 1990.
- At the same coefficient value, a forward looking Fed produces a larger correlation than a backward looking Fed.
  - This was a big surprise to me.
  - Perhaps I also need to explicitly model an improvement in forecasting quality.
    - Not sure how to do that in a model with rational expectations.



# Empirical Hybrid Taylor Rule

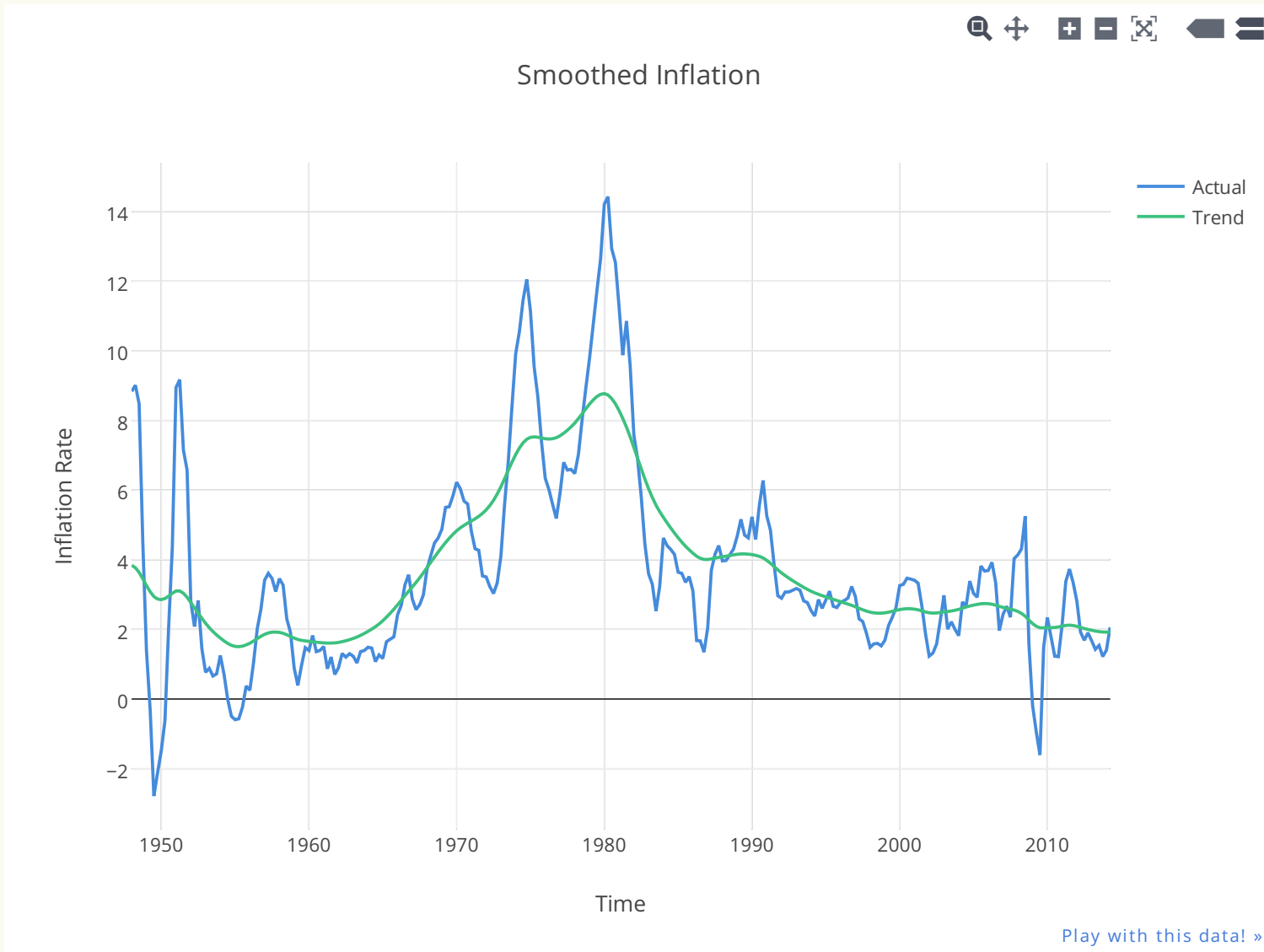
$$i_t - \pi_t = r^* + \rho_i i_{t-1} + \phi_{\pi_t} (\pi_t - \pi^T) + \phi_{\pi^e} (E_t \pi_{t+1} - \pi^T) + \phi_y (y_t - y^P)$$

- In order to estimate, need to find measures of inflation target, inflation expectations, and potential output.

# Inflation Target

- Use Bayesian estimation of local level model to extract trend from CPI data (similar procedure as Stock & Watson).
- Will eventually include stochastic volatility.

# Inflation Target



# Inflation Expectations and Potential Output

- For now, Michigan Survey for inflation expectations & CBO Potential output.
- Eventually use real-time Greenbook estimates from the Fed (these are slightly harder to obtain because they aren't on FRED)

# Preliminary Estimation

$$i_t - \pi_t = r^* + \rho_i i_{t-1} + \phi_{\pi_t} (\pi_t - \pi^T) + \phi_{\pi^e} (E_t \pi_{t+1} - \pi^T) + \phi_y (y_t - y^P)$$

For the period 1983-1992

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Parameter	Inclusion Probability
$r^*$	12.7%
$\rho_i$	100.0%
$\phi_{\pi_t}$	32.7%
$\phi_{\pi^e}$	13.1%
$\phi_y$	8.2%

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# Preliminary Estimation

$$i_t - \pi_t = r^* + \rho_i i_{t-1} + \phi_{\pi_t} (\pi_t - \pi^T) + \phi_{\pi^e} (E_t \pi_{t+1} - \pi^T) + \phi_y (y_t - y^P)$$

For the period 1992-2008

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Parameter	Inclusion Probability
$r^*$	10.4%
$\rho_i$	100.0%
$\phi_{\pi_t}$	96.3%
$\phi_{\pi^e}$	79.7%
$\phi_y$	23.7%

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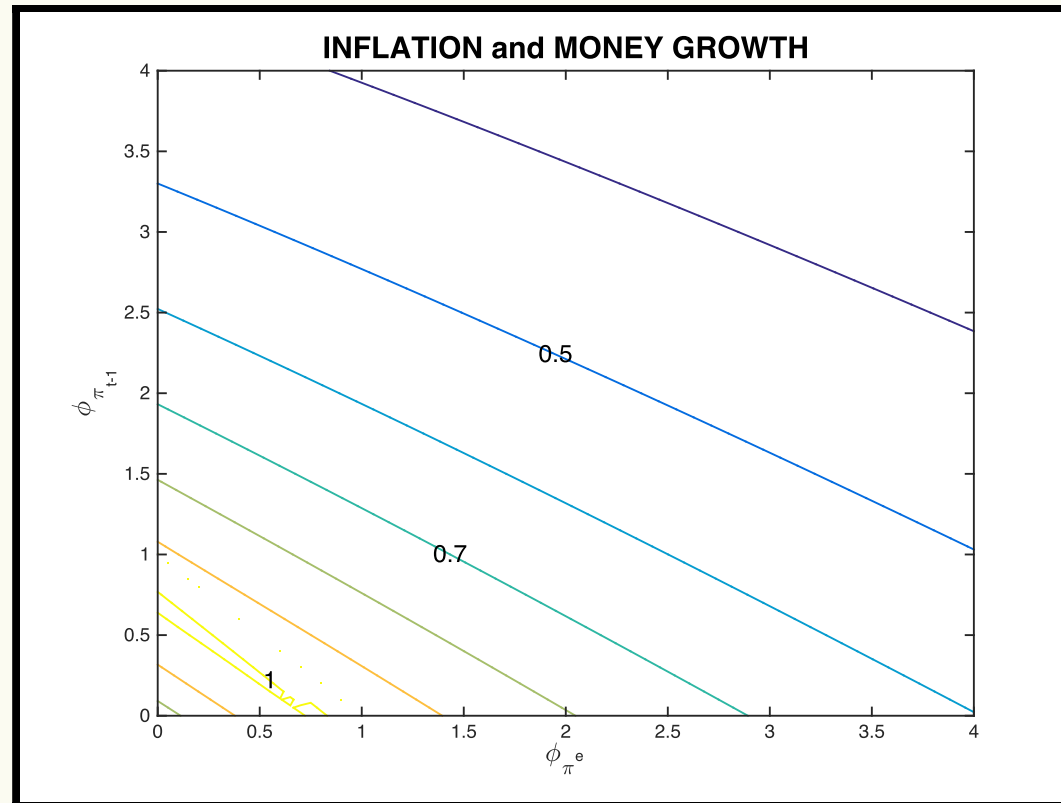
# Preliminary Theoretical

- Solve Sargent & Surico's model at their estimated parameters.

- Use a hybrid Taylor rule:

$$R_t = \rho_r R_{t-1} + (1 - \rho_r)(\phi_{\pi_{t-1}} \pi_{t-1} + \phi_{\pi^e} E_t \pi_{t+1}) + \varepsilon_{Rt}$$

# Preliminary Theoretical



Correlation & Hybrid Rule



# To Do

- Get Greenbook data.
- Code up time varying dimension model.
- Think a lot harder about theoretical model, and what improvements to Fed policy during the 1980s/1990s would have looked like (improvements in forecasting ability?)

# Summary

- Lots of recent work on the break-down of the quantity theory of money.
- Two main hypotheses:
  - Fed policy
  - Mismeasurement of money
- My idea:
  - Think along the lines of Sargent and Surico
  - How much of the declining correlation can be attributed to changes in Fed policy?
    - Use Bayesian estimation of TVD model to analyze changes in policy.
    - Use Theoretical model to investigate how these changes translate into changes in the correlation between money and output.