

Basic Electricity and Solving Circuit Problems

A circuit consists of basic elements (resistors, batteries, capacitors, etc.) connected together at nodes.

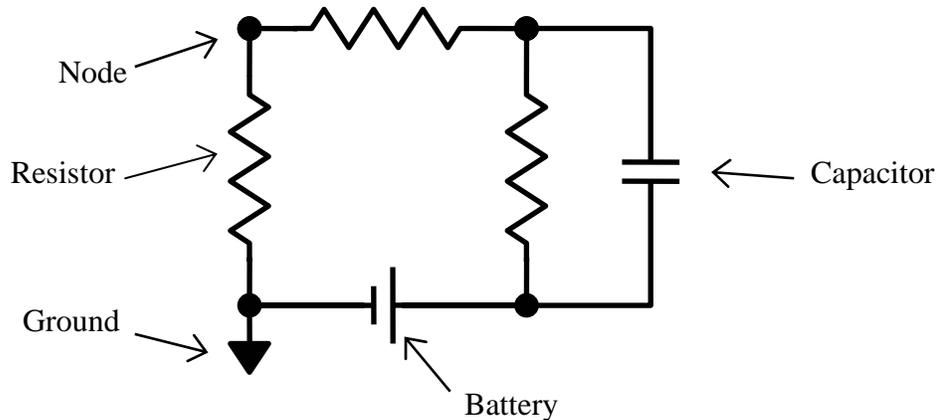


Figure 1. A simple circuit.

Most of the basic elements have two ends (2 terminals). Two common 2-terminal elements are resistors and capacitors:



Figure 2. Two common circuit elements

Three quantities can be measured at a 2-terminal element: the voltage at each end (V_{tail} , V_{head}) and the current passing through (I). The arrow defines the sign convention for the current flowing through the element (see below). V_{tail} is the voltage at the tail end of the arrow and V_{head} is the voltage at the head end. Each of the 3 quantities (V_{tail} , V_{head} , and I) can vary with time.

IMPORTANT NOTE: The current (I) that enters at one terminal exits unchanged at the other (conservation of charge). That's why it makes sense to talk about the current passing through the element. This is true of all 2-terminal devices, including capacitors. When people say that charge accumulates on a capacitor, what they really mean is that some current enters at one end and the same amount of current leaves at the other end. Since the current is blocked in the middle, positive charge accumulates on one plate and negative charge on the other plate, but the total charge is at all times equal to zero.

Ohm's Law. Ohm's law applies to an ideal resistor. It relates the current passing through the resistor to the voltage difference between the two ends. It is usually written: $V = I \cdot R$. However, when actually used to solve problems, the complete form needs to be used:

$$V_{\text{tail}} - V_{\text{head}} = I \cdot R.$$

An annoyance: In general, biologists (like engineers) are very sloppy about defining symbols. They almost never bother to tell you which symbols represent constants and which can vary. Instead of writing $V = I \cdot R$, it would be better to write $V(t) = I(t) \cdot R$, to make it clear that V and I vary with time, but R does not. Often, V and I vary with position as well. Sometimes, even R can vary with time and/or position. By sweeping these issues under the rug, biologists succeed in writing formulae that look neater but are more confusing.

Units: The magnitude of a current is measured in Amperes: 1 Ampere = 1 coulomb/s = 10.36 μ moles of positive unit charges/s. In other words, there are 10.36 μ moles of charge/coulomb, or equivalently, 96,500 coulombs/mole of charge. The number of coulombs per mole is called the Faraday: $F = 96500$ coulombs/mole of charge.

Sign convention: To define the meaning of "positive current" through a circuit element, one must decide which is the "positive direction". It doesn't matter which direction you chose, but you must stick to the definition throughout the analysis. To keep track of the convention, I usually draw an arrow next to each element showing the "positive direction". This gives an orientation to the element, and thus allows one to specify which voltage is V_{head} and which is V_{tail} . In the equation for Ohm's law, net positive charge flowing in the direction of the arrow (or negative charge flowing in the opposite direction) has a positive sign.

The modern sign convention for current across a cell membrane is that positive current means positive charge leaving the cell. In other words, by convention, outward membrane current is positive. Thus, potassium ions leaving the cell create a positive current. Chloride ions entering the cell also create a positive current. From the sign of the current, one cannot tell the direction that ions are moving unless one knows whether the ions are positive or negative.

The opposite convention is followed for current entering a cell through an electrode: positive charge flowing through the electrode into the cell is (usually) called a positive current. The reason for this convention is that charge entering a cell through an electrode causes an equal charge to leave the cell across the cell membrane. Thus, the positive current always indicates outward current flow across the cell membrane.

The Rule of Electrical Neutrality The statement that the current entering a cell through an electrode equals the current leaving through the membrane is a special case of a very important and powerful principle, the electrical neutrality of any volume of a conductive medium. This principle leads to the following statement:

The net electric charge of a cell is always zero.

But how can this be? Doesn't a cell with a negative resting potential have a net negative charge? Doesn't the influx of Na^+ ions during the action potential cause positive charge to accumulate in the cell? The confusion here comes from the failure to specify exactly where the border of the cell lies. When current flows across the cell membrane, the only place where net charge movement occurs is at the membrane (i.e., within a few tenths of a nanometer of the surface). For practical reasons, it is most convenient to include the entire cell membrane, including its outer surface and a few nanometers further, as part of the cell. Thus, there is no net gain or loss of charge from the

cell, only the movement of charge from one part (the outer surface of the membrane), to another (the inner surface). This does not mean that Na^+ ions entering a cell through Na^+ channels all end up plastered to the membrane. Many of them end up floating around inside the cell, and an equivalent number of other ions moves from the cell interior to the cell membrane (or, in the case of Cl^- , away from the membrane). In the usual diagrams that represent the cell membrane as a capacitor in parallel with one or more resistors, the principle of electrical neutrality means that sum of the currents through all of these parallel pathways is zero.

Review of current in capacitors

Symbols:

- C Capacitance (in farads, F). F is a unit of measurement, different from Faraday's constant that appears in the Nernst equation and below.
- Q Charge (in coulombs, coul) A coulomb is the quantity of charge transferred by 1 ampere in 1 second.
- F Faraday's constant: the number of coulombs of charge per mole of protons. $F = 96,500$ coul/mole. Therefore, 10 μmole of singly-charged cations (Na^+ , K^+ , etc.) has approximately 1 coul of charge.

A capacitor consists of 2 conductors separated by a thin insulator. When a voltage, V , is applied across the conductors, a charge Q is driven onto one plate, and $-Q$ onto the other plate. Note that the net charge on the capacitor ($Q-Q$) remains 0. The relationship between Q and V is:

$$Q = C \cdot V.$$

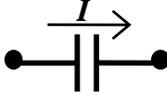
The current through the capacitor is:

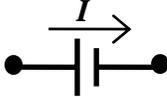
$$dQ/dt = I = C \cdot dV/dt$$

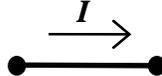
| name | parameter (units) | common symbols | equation |
|------|-------------------|----------------|----------|
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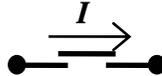
2-terminal elements:

| | | | |
|----------|-------------|--|---|
| resistor | R (Ohms) |  | |
| | | V_{tail} | $V_{\text{head}} \quad V_{\text{tail}} - V_{\text{head}} = I \cdot R$ |

| | | | |
|-----------|--------------|--|---|
| capacitor | C (Farads) |  | |
| | | V_{tail} | V_{head} |
| | | | $I = C \cdot d(V_{\text{tail}} - V_{\text{head}})/dt$ |

| | | | |
|---------|-------------------------------|--|---|
| battery | V_{battery} (Volts) |  | |
| | | V_{tail} | V_{head} |
| | | | $V_{\text{tail}} - V_{\text{head}} = V_{\text{battery}}$ (usually the longer plate is the positive terminal) |

| | | | |
|------|------|--|---|
| wire | none |  | |
| | | V_{tail} | V_{head} |
| | | | $V_{\text{tail}} - V_{\text{head}} = 0$ |

| | | | |
|--------|------|--|--|
| switch | none |  | |
| | | V_{tail} | V_{head} |
| | | | $V_{\text{tail}} - V_{\text{head}} = 0$ (closed) $I = 0$ (open) |

1-terminal element:

ground

none

V



$V = 0$
