Physics 610/Chemistry 678  
Summer 2015  
Problem Set #5  
Due Tuesday, July 28th

- Questions from Sze: 11.4, 11.5, 11.6, 14.1, 14.3  
- And the following questions:

1. Use the definition of the pumping speed

\[ S \equiv \frac{dV}{dt} \]

to derive an expression for the pump-down time of a vacuum system as a function of the chamber volume, the pumping speed at the end of the vacuum chamber, and the initial and final pressure of the vacuum chamber. Your vacuum pump has a pumping speed of \( S_p \) and its inlet is connected to your vacuum chamber by a stainless steel vacuum hose of diameter \( d \) and length \( L \). Modify your expression for the pump-down time to incorporate \( S_p, d, \) and \( L \) in the molecular flow and viscous flow regimes. Explain your result. How does one minimize the pumping time for a given pump speed?

2. The distance between source and wafer in a deposition chamber is 15 cm.
   a. Estimate the pressure at which this distance becomes 20% of the mean free path of source molecules.
   b. What is the impingement rate at this pressure? About how long does it take to form a monolayer of oxygen on the surface? (you can assume the gas is \( O_2 \) and that the sticking probability is 0.1)

3. Calculate the contrast and critical modulation transfer function (CMTF) for the following exposure response curves. Are the resists negative or positive? Order them in terms of their sensitivity.
4. (Sze 13.4)
   a. For an ArF excimer laser 193 nm optical lithographic system with $NA = 0.65$, $k_1 = 0.60$, and $k_2 = 0.50$, what are the theoretical resolution and depth of focus for this equipment?
   b. What can we do in practice to adjust $NA$, $k_1$, and $k_2$ parameters to improve resolution?
   c. What parameter does the phase-shift mask (PSM) technique change to improve resolution?

5. Numerical solutions to differential equations with Mathematica. For this problem, you will need to download and install a copy of Mathematica on your computer, or use one of the University’s computer workstations (in the Science Library, for example). Information for downloading Mathematica using UO’s site licensing is here: https://it.uoregon.edu/software/mathematica. Also, you will be expected to go through these exercises independently and to turn in any plots you generate with this problem set. You can, however, work together with others, but you need to execute the code, modify the code, and produce plots on your own.
   a. Some plotting basics. Paste the following code in a Mathematica notebook and press Shift + Enter on your keyboard to evaluate the code:

   ```mathematica
   Plot[Sin[x], {x, 0, 10}]
   ```

   How does changing the values in the braces change the plot? Now let’s use the Manipulate command, which allows you to explore how a parameter affects your result without rerunning code. Evaluate the following code:

   ```mathematica
   Manipulate[Plot[Sin[a*x + b], {x, 0, 10}], {a, 0, 10}, {b, 0, 2*Pi}]
   ```

   Adjust the toggle bar of the Manipulate GUI and click on the + sign at the end of the toggle bar to enable additional visualization features like parameter-sweep movies.

   b. Solving differential equations numerically with NDSolve. In class we discussed the heat conduction equation:

   $$\frac{\partial T(t, x)}{\partial t} = D \frac{\partial^2 T(t, x)}{\partial x^2}$$  \hspace{1cm} (1)

   This equation is equivalent to the diffusion equation. We used the heat conduction equation to explore the heating and cooling of the Earth due to diurnal and annual heating cycles caused by the Sun’s radiant energy. We will solve Eq. 1 using Mathematica with the following boundary conditions:

   $$T(t, 0) = \sin t$$
   $$T(0, x) = 0$$
   $$T(t, 100) = 0$$

   What do these boundary conditions mean?
   Evaluate the following code in Mathematica:

   ```mathematica
   a = 1;
b = 10;```
sol1 = NDSolve[\{D[u[t, x], t] == a*D[u[t, x], {x, 2}], u[0, x] == 0, u[t, 0] == Sin[t], u[t, 100] == 0\}, u, \{t, 0, 10\}, \{x, 0, 100\}];
sol2 = NDSolve[\{D[u[t, x], t] == b*D[u[t, x], {x, 2}], u[0, x] == 0, u[t, 0] == Sin[t], u[t, 100] == 0\}, u, \{t, 0, 10\}, \{x, 0, 100\}];
Manipulate[
  Plot[\{Evaluate[u[Time, x] /. sol1], Evaluate[u[Time, x] /. sol2]\}, \{x, 0, 10\}, PlotRange -> {-1, 1}, AxesLabel -> \{Depth, Temperature\}], \{Time, 0, 10\}]

In this code $T = u$, $D[u[t, x], t] = \frac{\partial T(t,x)}{\partial t}$ and $D[u[t, x], \{x, 2\}] = \frac{\partial^2 T(t,x)}{\partial x^2}$. What do the other parts of the code represent (e.g. what is $a$, what is $u[0, x] == 0$)? Discuss the results of the plotted solutions.

c. **EXTRA CREDIT**: Modify the above code to solve the problem where the thermal diffusion coefficient is a function of the temperature. Assume the dependence obeys a power law, i.e. $D(T) \propto T^\gamma$. How does the thermal profile change with $\gamma$? Plot and interpret your results.