Sediment disentrainment and the concept of local versus nonlocal transport on hillslopes

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[1] A local formulation of the sediment flux on a hillslope describes the flux as a unique function of local hillslope conditions at any contour position x, whereas a nonlocal formulation must take into account nonlocal (upslope or downslope) conditions that influence the flux at x. Local formulations are reasonable when particle motions involve small length scales associated with localized bioturbation of the soil column or with proximal surface transport such as rain splash. Nonlocal formulations may be more appropriate in steeplands where patchy, intermittent motions involve large travel distances, mostly over the surface. Once sediment motions are initiated, the disentrainment process determines the distribution of particle travel distances, which, in turn, forms the basis of nonlocal formulations that involve a convolution of hillslope surface conditions, for example, the land-surface slope. The kernel in the convolution integral, which weights the effect of land-surface conditions (e.g., slope) at all positions upslope or downslope of x, derives from the formulation of the disentrainment rate and characterizes whether particle travel distances depend on conditions at the position where motions originate or vary as particles experience changing surface conditions during their downslope motions. If hillslope properties controlling transport (e.g., surface slope) are defined or measured at a specified resolution, then motions smaller than this resolution cannot be attributed to these properties resolved at a smaller scale. In essence, the relative importance of local and nonlocal transport depends on the scale of particle motions compared to the relevant scale of hillslope properties that drive transport.

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1. Introduction

[2] Recent work on the topic of sediment transport on hillslopes suggests the need to distinguish between "local" and "nonlocal" transport processes and associated formulations of the sediment flux [*Foufoula-Georgiou et al.*, 2010; *Furbish and Haff*, 2010; *Tucker and Bradley*, 2010; *Gabet and Mendoza*, 2012]. The mathematical distinction between local and nonlocal transport is clear. Namely, the definition of local versus nonlocal transport centers on whether the sediment flux at a contour position x can be expressed as a unique function of local hillslope conditions at x (for example, the local land-surface slope), or whether the flux at x also depends on hillslope conditions a significant distance upslope or downslope of this position, in which case the flux at x must be expressed in a way that takes into account these nonlocal (upslope or downslope) conditions. A physical distinction between local and nonlocal transport is less clear, owing to the variety of processes that contribute to sediment transport on hillslopes and to the widely varying length scales of sediment motions during transport. Local transport generally involves sediment motions that are sufficiently small wherein their collective contribution to transport at a position x can be characterized in terms of local hillslope conditions at x. In contrast, nonlocal transport involves sediment motions that are sufficiently large that it becomes necessary to functionally relate these motions to upslope (or downslope) conditions, which may be different from those at x.

[3] For example, as summarized in *Furbish et al.* [2009a] and *Furbish and Haff* [2010], there is a long record of work suggesting that the rate of downslope transport of soil material by creep on soil-mantled hillslopes is approximately proportional to the local land surface slope. Inasmuch as soil creep involves the collective, quasi-random motions of soil particles associated with small-scale bioturbation, effects of wetting-drying [e.g., *Kirkby*, 1967] or freeze-thaw cycles [*Anderson*, 2002], where particle motions span pore to many-pore length scales during creation and collapse of

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porosity within the soil column, then a (local) linear relation between transport and land surface slope is well founded [*Furbish et al.*, 2009b]. Similarly, the downslope drift of soil particles associated with rain splash can be viewed as local transport, inasmuch as the sediment flux arising from this downslope drift is related to the local land-surface slope [*Furbish et al.*, 2007, 2009a; *Dunne et al.*, 2010]. Both of these examples by definition represent local formulations of transport with respect to slope.

[4] Particularly in steeplands, local formulations of transport may not adequately characterize transport behavior. With increasing steepness, soil motions associated with ravel [e.g., *Gabet and Mendoza*, 2012], soil slips, transport by fossorial animals [*Gabet*, 2000], or tree throw, [*Norman et al.*, 1995] and possibly transport by patchy, intermittent surface flows following fire [*Roering and Gerber*, 2005], can involve downslope travel distances that are much larger than those associated with small-scale bioturbation or freeze-thaw acting within the soil column, or those associated with rain splash [*Roering et al.*, 1999; *Foufoula-Georgiou et al.*, 2010]. In these cases, the downslope flux of soil material past a given contour position x can involve motions that originate from near to far upslope and traverse significant distances downslope of x before coming to rest. Key qualities of transport (e.g., the amount of soil material mobilized or the travel distances of soil particles) may depend on hillslope conditions at the site where the motions begin, which are different from those at x. Equally important, these motions are patchy and intermittent, and mostly involve dispersal of soil material over the land surface, where material moves rapidly in comparison with the slower bulk soil motion arising from creation and collapse of porosity. In this situation, an appealing approach is to describe the flux in terms of a convolution integral which in principle weights the effects of the conditions at all positions upslope and downslope of x that contribute to the flux at x [*Foufoula-Georgiou et al.*, 2010; *Furbish and Haff*, 2010]. This represents a nonlocal formulation of transport.

[5] The differences between local versus nonlocal transport have far-reaching implications for the evolution of hillslope topography. If, for example, the downslope soil flux at a position x is proportional to the local land-surface slope, then that part of the local rate of change in the surface elevation determined by the divergence of the flux at x is proportional to the local derivative of the slope, independent of upslope and downslope conditions. (Hereafter, we refer to the local derivative of the slope as the land-surface "concavity," carrying magnitude and sign). Moreover, this divergence is the same at any two sites with the same concavity. In contrast, if the soil flux at a particular hillslope position depends on the local and upslope conditions, then the rate of change in the surface elevation at two sites with identical local conditions (e.g., slope and concavity) but different upslope configurations might be entirely different.

[6] Because the topic of nonlocal transport, and its relation to local transport, is relatively new, there is uncertainty in the literature regarding the definitions and distinguishing features of local versus nonlocal formulations of transport. Our first objective therefore is to step through the essential elements that characterize local versus nonlocal formulations. We then focus on the idea of sediment disentrainment following mobilization—how sediment particles come to rest after traveling some distance downslope (or upslope) in relation to land-surface conditions experienced by the particles during their motions. Indeed, the disentrainment process has a central role in determining the distribution of particle travel distances [*Furbish and Haff*, 2010; *Furbish et al.*, 2012; *Roseberry et al.*, 2012], which, in turn, forms the basis of the (nonlocal) convolution form of the sediment flux. We specifically demonstrate how a probabilistic description of sediment disentrainment represents a unifying physical basis for explaining how different distributions of particle travel distances naturally arise from assumptions regarding their travel and disentrainment—from the scale of particle trajectories resulting from raindrop impacts to the scale of particle motions that approach the full length of a hillslope. Our description of nonlocal transport therefore goes far beyond recent treatments of this topic, which are limited to specific cases of particle behavior [*Foufoula-Georgiou et al.*, 2010; *Furbish and Haff*, 2010; *Tucker and Bradley*, 2010; *Gabet and Mendoza*, 2012].

[7] In section 2, we qualitatively illustrate the ideas of local and nonlocal transport, focusing on the functional relation of the flux to land-surface slope. This section provides a concise description of the essence of what distinguishes nonlocal from local formulations of transport. In section 3, we provide a definition of the sediment flux on hillslopes, formulating it as a quantity that is averaged over space and time in a way that takes into account the patchy, intermittent sediment motions that characterize many transport processes in steeplands. We then show how the flux, expressed as a convolution integral, formally derives from the distribution of particle travel distances, and how the quantity being convolved consists of the sediment mobilization rate. In section 4, we formulate the disentrainment rate function and its relation to the distribution of particle travel distances, and we present an example involving the effects of land-surface slope on disentrainment. Here we illustrate how the distribution of travel distances may be determined by conditions at the origin of motion or, alternatively, how this distribution may be altered due to the fact that particles experience changing conditions during their downslope motions. In section 5, we return to the definition of the sediment flux expressed in the form of a convolution integral and show how the flux varies depending on the form of the distribution of travel distances and the underlying description of disentrainment rates. We then show how the convolution can be recast in the form of an advection-diffusion equation in the case where the distribution of travel distances possesses finite mean and variance. In section 6, we compare the convolution-integral form of the flux with the advection-diffusion approximation of this convolution and illustrate how both yield a nonunique relationship between the sediment flux and the local land-surface slope. This suggests that the evolution of hillslope profiles involving nonlocal versus local transport are likely distinct. We then show that the advection-diffusion approximation involving the land-surface slope may be reinterpreted as a local formulation if the idea of a local formulation is generalized to include more than the land-surface slope. In section 7, we return to the physical distinction between local and nonlocal transport. Because the (nonlocal) convolution form of the flux is quite general, this distinction hinges on the

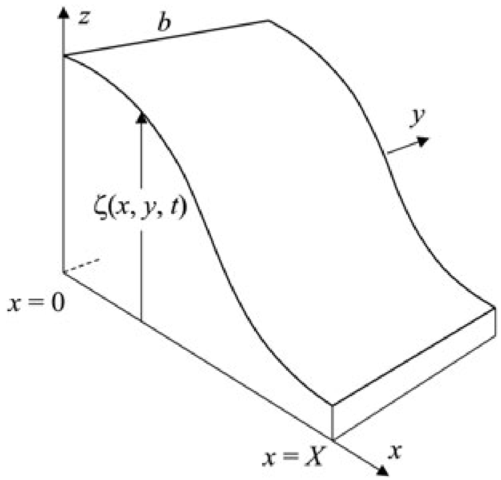

Figure 1. Definition diagram showing a convex-concave hillslope and associated xyz coordinate system, with hillslope length X, width b, and local land-surface elevation $\zeta(x, y, t)$.

scale of particle motions relative to the scale of resolution at which the factors controlling transport are defined or measured.

2. Local Versus Nonlocal Transport

[8] The concept of local versus nonlocal transport can be concisely illustrated by considering an idealized situation in which the sediment flux is related to a single fundamental quality of a hillslope profile, namely, its local slope. Consider a convex-concave hillslope (Figure 1), and let x, y, and z [L] denote associated Cartesian coordinates. The horizontal x axis is positive in the downslope direction with origin ($x = 0$) at the hillslope crest. The foot of the hillslope is positioned at $x = X$. The horizontal y axis is positive toward the left when looking downslope, with origin ($y = 0$) along the hillslope "axis." For later reference, a convenient hillslope "width" b [L] is defined by $y = \pm b/2$. The z axis is positive upward. Letting t [T] denotes time, then $z = \zeta(x, y, t)$ [L] denotes the local elevation of the land surface.

[9] Focusing momentarily on one-dimensional transport parallel to x, consider a land-surface profile $\zeta(x, t)$ along the axis of the convex-concave hillslope (Figure 2). Let us now assume that the flux of soil material $q(x, t)$ [L^2 T^{-1}] at a position x and time t is only a function of the slope $S(x, t) = \partial\zeta(x, t)/\partial x$, and possible other quantities $\mathbf{H}(x, t)$, defined locally at this position, namely,

$$q(x, t) = f[S(x, t), \mathbf{H}(x, t)] . \quad (1)$$

Here $\mathbf{H}(x, t)$ denotes a vector of quantities, for example, the local soil thickness, the rate at which soil particles are mobilized, the soil particle size, and so on. Let us now choose two positions x on the hillslope profile, x_1 and x_2, that possess the same local slope $S(x, t)$, that is, $S(x_1, t) = S(x_2, t)$. Then, at any time t, the flux $q(x, t)$ at the two positions x_1 and x_2 is the same for $\mathbf{H}(x_1, t) = \mathbf{H}(x_2, t)$. This is true whether the flux possesses a simple linear dependence on slope [*Culling*, 1963, 1965; *Kirkby*, 1967; *Carson and Kirkby*, 1972; *Hirano*, 1975; *Nash*, 1980a, 1980b; *McKean et al.*, 1993; *Fernandes and Dietrich*, 1997; *Martin and Church*, 1997; *Heimsath et al.*, 1999], namely,

$$q(x, t) = -DS(x, t) , \quad (2)$$

or whether it might possess a nonlinear dependence on slope [e.g., *Andrews and Bucknam*, 1987; *Roering et al.*, 1999], for example,

$$q(x, t) = -D\frac{S}{1 - (|S|/S_c)^2} , \quad (3)$$

or a nonlinear dependence involving the product of soil thickness and slope [e.g., *Ahnert*, 1967; *Anderson*, 2002; *Furbish et al.*, 2009a], for example,

$$q(x, t) = -Kh(x, t)S(x, t) , \quad (4)$$

where D [L^2 T^{-1}] represents a diffusion-like coefficient, S_c denotes a critical slope, K [L T^{-1}] represents a transport coefficient, and $h(x, t)$ [L] is the local soil thickness. The same idea holds for positions with the same slope $S(x, t)$ on different hillslopes subjected to the same environmental conditions. In this respect, (2), (3), and (4) each represents a "local" formulation of transport, specifically with respect to land-surface slope. Note also that, as described in section 3.1, although the flux is written here as an "instantaneous" quantity, we must envision this as being averaged over fluctuations that occur at timescales much less than the mean residence time of soil particles on a hillslope. Conceptually, this is equivalent to envisioning the flux as varying smoothly in time when viewed at timescales longer than the mean residence time.

[10] Now consider an interesting alternative. Again, choose two positions x on the profile, x_1 and x_2, that possess the same local slope, namely, $S(x_1, t) = S(x_2, t)$ (Figure 2). Observe that the serial configurations of $S(x, t)$ upslope of the two positions differ. At the higher position x_1, the upslope configuration of the profile is convex. At the lower position x_2, the upslope configuration of the profile is concave nearby and convex farther away. Suppose that material crossing x per unit time originates from "far" upslope. Equally important, suppose that the amount of material

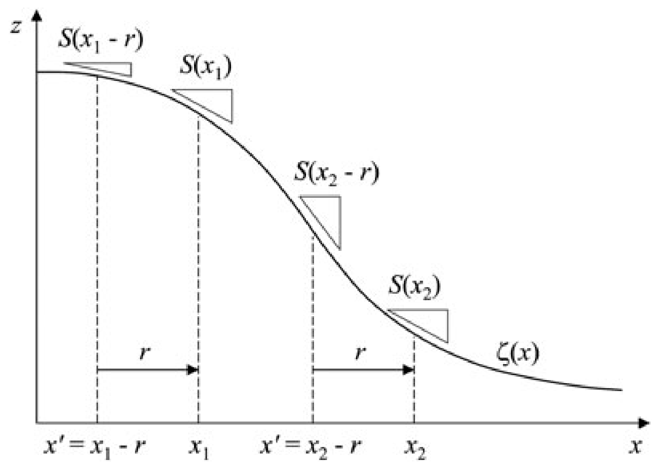

Figure 2. Definition diagram showing land-surface profile $\zeta(x)$ and positions x_1 and x_2 with same local slope $S(x_1) = S(x_2)$ represented by triangles, and different slopes $S(x_1 - r) \neq S(x_2 - r)$ at a distance r upslope.

crossing x from any upslope position x' depends on the slope $S(x', t)$ at this upslope position. Thus, the material crossing position x from upslope does not depend just on the local slope at x but rather also on the slope at $x' = x - r$. That is, the flux at x with slope $S(x, t)$ depends in part on the slope $S(x', t) = S(x - r, t)$. For a position x' at a given distance r upslope from x_1 and x_2, the slope is different; so, the amount of material delivered to x_1 from $x_1 - r$ is different than the amount delivered to x_2 from $x_2 - r$. The same conclusion applies to all other positions x' at different upslope distances r. Thus, the flux at x_1 must be different than the flux at x_2, despite identical local slopes at these two positions. This defines the essence of nonlocal transport. Namely, a formulation of the flux at any x must take into account the serial configuration of upslope conditions. As elaborated in later sections, one way to describe this condition is to write a convolution of the form [e.g., *Foufoula-Georgiou et al.*, 2010; *Furbish and Haff*, 2010]

$$q(x, t) = \int_{-\infty}^{x} h_r(x - x') f[S(x', t)]\, \mathrm{d}x' , \tag{5}$$

where $h_r(x{-}x')$ is a kernel that weights the effect of the land-surface slope at all positions upslope of x. This represents the idea of a "nonlocal" formulation of transport, specifically with respect to land-surface slope.

[11] We now show, starting in the next section, how the kernel $h_r(x{-}x')$ is obtained from a probabilistic description of the downslope travel distances r of sediment particles whose motions start from positions $x' = x - r$. This kernel may be a function of land-surface slope and other factors. We also show that the quantity in (5) being convolved, namely $f(x', t)$, is the rate at which sediment is mobilized. This too may be a function of surface slope. The formulation therefore provides a description of the physical ingredients of (5). We start with a definition of the sediment flux.

3. Definition of the Sediment Flux

3.1. Basic Definition

[12] With reference to Figures 1 and 2, consider particle motions that originate within the small interval x' to $x' + \mathrm{d}x'$ over a contour width b. Choosing for illustration the raveling process, these motions are intermittent and discontinuous (patchy) over the width b. Individual raveling events may involve few, or many, particles. The distribution of travel distances is likely to differ from one event to the next, and the average travel distances associated with individual events may be small or large, depending on the surface conditions during the events. In turn, these individual events contribute differing amounts of sediment to the total amount crossing a downslope position x during an interval T. For these reasons, it is important to define the flux at x such that it represents a quantity that is averaged over space and time in a way that takes into account this patchy, intermittent behavior.

[13] Let x denotes a coordinate axis along which a sediment particle, starting from a position x', travels a distance $r = x - x'$ during an unspecified interval of time. The travel distance r is measured from start to stop, defining one travel "event," as opposed to representing multiple travel events with intervening rest periods. Consider, then, the travel distances of a great number of particles starting from x'. We may suppose that the initial conditions of particle motions (e.g., the initial velocities) vary from one particle to the next and that each particle experiences a unique set of conditions during its motion (e.g., interactions with the surface over which it travels), such that the travel distance r may be considered a random variable.

[14] Let $f_r(r; x')$ [L^{-1}] denotes the probability density function of the travel distance r of sediment particles whose motions begin at position x', that is, within the interval x' to $x' + \mathrm{d}x'$. We then envision the travel distances within the interval r to $r + \mathrm{d}r$ represented by the probability $f_r(r; x')\mathrm{d}r$ as deriving from the amalgamation of the travel distances r to $r + \mathrm{d}r$ involving many raveling events during T—as if all particles from all events were set in motion at the same instant. That is, $f_r(r; x')$ represents the distribution of travel distances that emerges during T. Alternatively, we may envision individual events as representing "samples" drawn from the distribution $f_r(r; x')$ during T.

[15] Conceptually, the interval T must be sufficiently long that the form of $f_r(r; x')$ becomes invariant with increasing time greater than T but shorter than the timescale over which significant changes in hillslope geometry occur. *Furbish and Haff* [2010] suggest that this is on the order of the mean residence time of soil particles on the hillslope. The timescale T, moreover, is not independent of the contour width b. Namely, for a given characteristic frequency and spatial patchiness of disturbances leading to ravel, the likelihood that material will be mobilized within the area $b\mathrm{d}x$ during a specified interval increases with increasing width b. In turn, the interval T required for $f_r(r; x')$ to become invariant presumably decreases with increasing b. However, b cannot be too large. Inasmuch as the distribution $f_r(r; x')$ is to be functionally related to local hillslope conditions, say, the local land-surface slope, then b must be smaller than the characteristic distance over which the land-surface slope significantly changes along contour [*Furbish and Haff*, 2010].

[16] The instantaneous, vertically integrated flux $q(x, t)$ [$\mathrm{L}^2\ \mathrm{T}^{-1}$] associated with a vertical surface A [L^2] of contour width b at position x is precisely defined as the surface integral of surface-normal particle velocities, namely,

$$q(x, t) = \frac{1}{b} \int_A \mathbf{u}_p \cdot \mathbf{n}\, \mathrm{d}A , \tag{6}$$

where $\mathbf{u}_p$ [$\mathrm{L}\ \mathrm{T}^{-1}$] is the discontinuous particle velocity field viewed at the surface A, $\mathbf{n}$ is the unit vector normal to A, and A extends over the vertical domain of moving particles. Here we note that, alternatively, if one starts with a volumetric flux [$\mathrm{L}\ \mathrm{T}^{-1}$] and vertically integrates this flux over the active soil thickness, then a length dimension is added, and the result is a solid volumetric discharge per unit width per unit time, that is, the sediment flux $q(x, t)$. But whereas this procedure is nominally the same as integrating the surface-normal particle velocity field $u_p = \mathbf{u}_p \cdot \mathbf{n}$ over the area A then dividing by the width b (as above), it is conceptually different, as it assumes a continuum framework from the outset—something we are purposefully avoiding here. Indeed, having the width b explicitly in the definition (6) emphasizes the fact that the width b must be a key part of the conceptualization of the flux, in that a conventional continuum approximation is not satisfactory, and the instantaneous flux varies with the width b [*Furbish et al.*, 2012].

[17] Letting t_0 denotes an arbitrary initial time, we now define the solid volumetric discharge at the position x as

$$Q(x,T) = \frac{b}{T}\int_{t_0}^{t_0+T} q(x,t)\,\mathrm{d}t\,, \tag{7}$$

which is an average over T. In turn, $q(x) = Q(x,T)/b$. In addition, in the formulations below, we consider the time-averaged mobilization rate of soil material. Namely, let $E(x', t)$ [L T^{-1}] denotes the instantaneous solid volume of soil material that is mobilized within $b\mathrm{d}x'$ per unit area per unit time. The time-average mobilization rate is then

$$E(x') = \frac{1}{T}\int_{t_0}^{t_0+T} E(x', t)\,\mathrm{d}t\,. \tag{8}$$

Hereafter, in neglecting the time dependence, we are assuming quasi-steady conditions over an averaging period T, analogous to the quasi-steady approximation used in treating turbulent flows [e.g., *Furbish*, 1997, p. 353].

3.2. The Flux in Convolution Form

[18] For simplicity, we consider particle motions only in the positive x (downslope) direction, then generalize to the case of bidirectional motions in section 6. Recalling that $f_r(r;x')$ is the probability density function of the travel distance r of sediment particles whose motions begin at position x', then by definition, the cumulative distribution function is

$$F_r(r;x') = \int_0^r f_r(w;x')\,\mathrm{d}w\,. \tag{9}$$

That is, $F_r(r;x')$ is the probability that a particle will experience a travel distance less than or equal to r. We now define the so-called "survival" function as $R_r(r;x') = 1 - F_r(r;x')$, which is the probability that a particle will travel (survive) a distance r or greater.

[19] If $E(x')$ denotes the (average) volume of soil material that is mobilized at x' per unit area per unit time, then $bE(x')\mathrm{d}x'\mathrm{d}t$ is the volume of material that is on average mobilized within the small interval $\mathrm{d}x'$ during $\mathrm{d}t$. The volume of soil particles starting from a position $x' < x$ and passing position $x = x' + r$ is $bE(x')R_r(x - x';x')\mathrm{d}x'\mathrm{d}t$, so the total volume passing position x during $\mathrm{d}t$ from all upslope positions is

$$Q(x)\,\mathrm{d}t = b\,\mathrm{d}t\int_{-\infty}^{x} E(x')R_r(x - x';x')\,\mathrm{d}x'\,. \tag{10}$$

Dividing by $b\,\mathrm{d}t$ then gives the volumetric flux $q(x)$ at position x, namely,

$$q(x) = \int_{-\infty}^{x} E(x')R_r(x - x';x')\,\mathrm{d}x'\,. \tag{11}$$

At this juncture, the form of the survival function $R_r(r;x')$ is arbitrary. Furthermore, the form of (11) is applicable across scales, from the scale of particle trajectories resulting from raindrop impacts to the scale of particle motions that approach the full length of a hillslope.

[20] We now show that the form of the distribution of travel distances, and thus the survival function $R_r(r;x')$, is fundamentally determined by the probabilistic nature of the sediment disentrainment process. The formulation begins with a consideration of sediment disentrainment rates.

4. Sediment Disentrainment

4.1. Disentrainment Rate Function

[21] Consider the travel distances of a great number of particles starting from x', initially without reference to the specific process producing the particle motions, nor to the details of the physics that bring the particles to rest. Recalling that $f_r(r;x')$ denotes the probability density function of the travel distance r of sediment particles whose motions begin at position x', we may then define a spatial disentrainment rate $P_r(r;x')$ [L^{-1}] as [*Furbish and Haff*, 2010; *Furbish et al.*, 2012]

$$P_r(r;x') = \frac{f_r(r;x')}{1 - F_r(r;x')} = \frac{f_r(r;x')}{R_r(r;x')}\,. \tag{12}$$

Note that $P_r(r;x')\mathrm{d}r$ is a conditional probability. Namely, $P_r(r;x')\mathrm{d}r = f_r(r;x')\mathrm{d}r/R_r(r;x')$ represents the probability that a particle will come to rest—that is, become disentrained—within the interval r to $r + \mathrm{d}r$, given that it moved to a distance ("survived") at least as far as r. In other words, if N denotes the total number of particles, then $Nf_r(r;x')\mathrm{d}r$ represents the number of particles disentrained within the interval r to $r + \mathrm{d}r$ and $NR_r(r;x')$ is the number of particles that have not yet been disentrained over the distance r. Then, $P_r(r;x')\mathrm{d}r = Nf_r(r;x')\mathrm{d}r/NR_r(r;x') = f_r(r;x')\mathrm{d}r/R_r(r;x')$ is the proportion of the $NR_r(r;x')$ particles disentrained within r to $r + \mathrm{d}r$ and $P_r(r;x') = f_r(r;x')/R_r(r;x')$ is this disentrained proportion per unit distance. Thus, we are using "spatial disentrainment rate" to refer to a quantity of particles per unit distance rather than the usual reference to a quantity per unit time, as these are mathematically homologous. (If r instead represented the particle travel time, then P_r would be a temporal disentrainment rate) [*Furbish et al.*, 2012; *Roseberry et al.*, 2012].

[22] By definition, $f_r(r;x') = \mathrm{d}F_r(r;x')/\mathrm{d}r = -\mathrm{d}R_r(r;x')/\mathrm{d}r$, so using (12),

$$P_r(r;x') = -\frac{1}{R_r(r;x')}\frac{\mathrm{d}R_r(r;x')}{\mathrm{d}r}\,. \tag{13}$$

Integrating (13) then retrieves the probability density function $f_r(r;x')$ expressed in terms of the disentrainment rate $P_r(r;x')$, namely,

$$f_r(r;x') = P_r(r;x')\,e^{-\int_0^r P_r(w;x')\,\mathrm{d}w}\,. \tag{14}$$

This immediately indicates that the form of $f_r(r;x')$ in principle can be obtained if the disentrainment rate $P_r(r;x')$ is specified. Also note that, because $R_r(r;x') = f_r(r;x')/P_r(r;x')$,

$$R_r(r;x') = e^{-\int_0^r P_r(w;x')\,\mathrm{d}w}\,. \tag{15}$$

Thus, the survival function $R_r(r;x')$ in the definition of the flux (11) is physically determined by the nature of the disentrainment rate $P_r(r;x')$. Furthermore, as illustrated in the next section, the survival function is the same as the kernel $h_r(x - x')$ in the convolution (5), where the quantity being convolved consists of the sediment mobilization rate.

4.2. An Example Involving Land-Surface Slope

[23] Let us consider how the disentrainment rate $P_r(r;x')$ might vary with hillslope position as a function of the

intrinsic mechanical behavior of the particles during motion, or as a function of external conditions that influence this motion, or a combination of both. To do this, we use for illustration the formulation of the disentrainment rate provided by *Furbish and Haff* [2010]. The objective at hand is not to elaborate the physics of disentrainment, but rather, to examine the functional form of the disentrainment rate $P_r(r; x')$ and the associated distribution of travel distances $f_r(r; x')$ as these might depend on spatial variations in hillslope conditions, leading to the ingredients of nonlocal transport. That is, in stepping through the examples below, we show how different forms of the distribution of travel distances naturally arise from various assumptions regarding particle disentrainment in response to land-surface slope. For simplicity, we consider particle motions only in the positive x direction, and we neglect variations with respect to time. The formulation therefore pertains to extant hillslope conditions at a given time.

[24] If $\zeta(x)$ now denotes the local land-surface elevation of a hillslope and $S = \partial\zeta/\partial x$ denotes the local land-surface slope, then recalling that $x = x' + r$, the disentrainment rate may be related to the slope as [*Furbish and Haff*, 2010]

$$P_r(r; x') = \frac{1}{\lambda_0}\left[\frac{2S_c}{S_c - S(r; x')} - 1\right] \qquad |S| < S_c\,, \tag{16}$$

where λ_0 is a characteristic (e.g., average) length scale of particle motions on a horizontal surface ($S = 0$), and S_c is the magnitude of a critical slope beyond which particles in motion continue their motion indefinitely. Note that the land-surface slope S carries sign, so in a right-handed coordinate system, $S < 0$ where $\zeta(x)$ is decreasing with increasing x. Thus, with $S = 0$, $P_r = 1/\lambda_0$, and when $|S| \to S_c$, $P_r \to 0$. As described in *Furbish and Haff* [2010], this formulation of $P_r(r; x')$ appeals to the idea that land-surface slope must enter the problem of particle disentrainment at lowest order. That is, although the formulation does not explicitly involve particle momentum [e.g., *Gabet and Mendoza*, 2012], it implicitly involves a friction-like behavior [e.g., *Gabet*, 2003] inasmuch as the dispersal length scale decreases with decreasing steepness. The critical slope S_c nominally coincides with the critical slope introduced by *Roering et al.* [1999], and with the friction angle introduced by *Gabet* [2003] for the specific case of dry ravel. Moreover, the formulation is entirely consistent with rain splash transport [*Furbish et al.*, 2007, 2009a; *Dunne et al.*, 2010], wherein λ_0 is finite on a horizontal surface and $P_r(r; x')$ varies approximately linearly with surface slope.

[25] Consider first the case of a uniformly rough, planar hillslope surface with constant slope S. The travel distances r depend on the intrinsic mechanical behavior of particles, as they interact with the surface during their downslope motions, akin to the experiments of *Gabet and Mendoza* [2012]. In this situation, (16) becomes

$$P_r = \frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S} - 1\right)\,, \tag{17}$$

so that (14) becomes

$$f_r(r) = \frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S} - 1\right) e^{-\frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S} - 1\right)\int_0^r \mathrm{d}w}\,. \tag{18}$$

Evaluating the integral then leads to an exponential distribution of travel distances, namely,

$$f_r(r) = \frac{1}{\mu_r} e^{-r/\mu_r} \qquad |S| \le S_c\,, \tag{19}$$

where $\mu_r = \lambda_0[2S_c/(S_c - S) - 1]^{-1}$ is the average travel distance. Thus, in this example, the probability of disentrainment is constant for a given land-surface slope. The average travel distance varies with the magnitude of the surface slope inasmuch as the slope influences particle motions and the disentrainment process. The form of the distribution of travel distances, $f_r(r)$, and the average travel distance, μ_r, are independent of the starting position x' on the planar surface. Although the exponential distribution by definition possesses a "normal" tail, in the limit of $|S| \to S_c$, both the average travel distance and the variance of the travel distance become undefined; particles are not disentrained downslope of the starting position.

[26] Suppose that the land-surface slope varies with hillslope position, that is, $S = S(x')$. To illustrate a specific case, let $S(x') = S_0 + \beta x'$. For example, with $S_0 = 0$ and $\beta < 0$ [L^{-1}], the land-surface elevation $\zeta(x')$ is parabolic and convex [e.g., *Fernandes and Dietrich*, 1997]. If $S_0 < 0$ and $\beta > 0$, then the land-surface elevation is parabolic and concave. Now (16) looks like

$$P_r = \frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S_0 - \beta x'} - 1\right)\,, \tag{20}$$

so that (14) becomes

$$f_r(r; x') = \frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S_0 - \beta x'} - 1\right) e^{-\frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S_0 - \beta x'} - 1\right)\int_0^r \mathrm{d}w}\,. \tag{21}$$

Evaluating the integral again leads to an exponential distribution $f_r(r; x')$ with $\mu_r(x') = \lambda_0[2S_c/(S_c - S_0 - \beta x') - 1]^{-1}$. In this situation, the probability of disentrainment is a constant value that depends on the local slope at the starting position x'. That is, the exponential form of the distribution of particle travel distances is invariant with hillslope position x', but the average travel distance varies with the starting position x'. For example, on a convex hillslope, particles starting from a position x' low on the hillslope see a steeper surface than do particles starting from a position x' higher on the hillslope, and the average travel distance of particles starting from the low position is greater than the average travel distance of particles starting from the higher position (Figure 3). Nonetheless, particle motions starting from any x', as characterized by $f_r(r; x')$, are conditioned only on the "local" slope at the starting position x' and are unaffected by the increasing steepness experienced during their motions downslope. This approximation is reasonable only if the average travel distance is much less than a characteristic length scale defined by $\lambda_S = |S|/(\mathrm{d}|S|/\mathrm{d}x)$ [*Furbish and Haff*, 2010], which characterizes the distance over which the magnitude of the land-surface slope significantly changes (or alternatively, does not change). That is, with $\mu_r \ll \lambda_S$, particles do not experience any significant change in the land-surface slope during their motions downslope from their starting position. Also, as in the case above, in the limit of $|S| \to S_c$, particles are not disentrained downslope.

[27] Suppose, then, that we write $S(x) = S_0 + \beta x$, indicating as above that the land-surface slope varies linearly with hillslope position. Note that this is the same as writing $S(x') = S_0 + \beta(x' + r)$. This opens the possibility that particle motions depend on the local land-surface slope at the starting position x' and also on changing slope conditions encountered downslope of x' during movement over the distance r. Now (16) looks like

$$P_r = \frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S_0 - \beta x' - \beta r} - 1\right), \tag{22}$$

so that (14) becomes

$$f_r(r; x') = \frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S_0 - \beta x' - \beta r} - 1\right) \cdot e^{-\frac{1}{\lambda_0}\int_0^r\left(\frac{2S_c}{S_c - S_0 - \beta x' - \beta r} - 1\right)\mathrm{d}w}. \tag{23}$$

To simplify the notation, let $B(x') = S_c - S_0 - \beta x'$. Evaluating the integral then leads to

$$f_r(r; x') = \frac{1}{\lambda_0}\left[\frac{2S_c}{B(x') - \beta r} - 1\right]\left[\frac{B(x')}{B(x') - \beta r}\right]^{-\frac{2S_c}{\lambda_0 \beta}} e^{r/\lambda_0}. \tag{24}$$

In this situation, the form of $f_r(r; x')$ depends not just on the starting position of particles x' but also on the serial configuration of land-surface slope downslope of the starting position. Thus, particle motions starting at any position x', as characterized by $f_r(r; x')$, are conditioned "nonlocally" by the changing surface slope during their motions downslope. For example, with increasing steepness downslope, particles see this steepening during their motions, whence the probability of disentrainment decreases, and the particles become dispersed a greater distance downslope than they otherwise would be if their motions were conditioned only by the slope at the starting position, as in the preceding example (Figure 3). Conversely, with decreasing steepness downslope, the probability of disentrainment increases, and the particles become dispersed over a shorter distance downslope than they otherwise would be if their motions were conditioned only by the slope at the starting position. This is like the behavior of the probabilistic particle model of *Tucker and Bradley* [2010], wherein the likelihood of continued downslope motion (or disentrainment) depends on the local surface conditions encountered during particle motion. Moreover, this changing likelihood of disentrainment is consistent with experiments involving particle transport over a rough surface, where, with increasing steepness, particle-surface interactions (i.e., momentum exchanges) change, so the probability of disentrainment changes, and concomitantly, the distribution of travel distances changes [*Gabet and Mendoza*, 2012].

[28] We note that (24) cannot be integrated analytically, so we cannot write an analytical expression for the average travel distance or the variance of this distribution. Nonetheless, numerical integration indicates that both quantities are finite. As a point of reference, a distribution possesses a heavy right tail if

$$\lim_{r\to\infty} R_r(r; x')e^{\gamma r} = \infty \tag{25}$$

for all $\gamma > 0$. Indeed, evaluating (25) with the survival function associated with (24) indicates that this condition is not satisfied for sufficiently small γ.

[29] Consider now the situation in which the disentrainment rate varies with the travel distance r regardless of changing surface conditions downslope of the starting position. To do this, we choose a disentrainment rate associated with a heavy-tailed distribution. For example, suppose that

$$P_r(r) = \frac{\alpha}{r_0 + r}, \tag{26}$$

where α is a shape parameter and r_0 is a location parameter. This describes a decreasing rate of disentrainment with increasing travel distance r. That is, the farther a particle travels, the less likely it is to stop. As written, (26) is independent of position x' (and x), and it may therefore be considered as applying to a given constant land-surface slope. Now (14) becomes

$$f_r(r) = \frac{\alpha}{r_0 + r}\, e^{-\alpha\int_0^r \frac{1}{r_0 + w}\mathrm{d}w}. \tag{27}$$

Evaluating the integral then leads to

$$f_r(r) = \frac{\alpha r_0^{\alpha}}{(r_0 + r)^{\alpha+1}}, \tag{28}$$

which is a Pareto Type II (or Lomax) distribution with mean $\mu_r = \alpha r_0/(\alpha - 1)$ for $\alpha > 1$ and variance $\sigma_r^2 = \alpha r_0^2/[(\alpha - 1)^2(\alpha - 2)]$ for $\alpha > 2$. Thus, for $1 < \alpha < 2$, the mean is finite, but the variance is infinite. For $\alpha > 2$, both the mean and variance are finite.

[30] For later reference, let us heuristically assume that

$$\alpha = \left(\frac{S_c}{|S|}\right)^{\eta}, \tag{29}$$

where the exponent η determines the rate of change in α as the slope $|S|$ changes. With decreasing slope $|S|$, α becomes large, and the average travel distance approaches r_0. In the limit of $|S| \to S_c$, $\alpha \to 1$, whence the average travel distance and the variance become undefined. In turn, if $|S| = |S_0 + \beta x'|$, then (26) becomes

$$P_r(r; x') = \left(\frac{S_c}{|S_0 + \beta x'|}\right)^{\eta} \frac{1}{r_0 + r}. \tag{30}$$

As in the example above, the disentrainment rate is conditioned by the land-surface slope at the starting position x'. And, whereas disentrainment depends on the travel distance r beyond x', it does not explicitly depend on the changing slope S with increasing r. Substituting (30) into (14) and evaluating the integral again leads to a Pareto distribution with α given by (29) and average travel distance $\mu_r(x') = [S_c/(|S_0 + \beta x'|)]^{\eta} r_0/([S_c/(|S_0 + \beta x'|)]^{\eta} - 1)$ for $\alpha = [S_c/(|S_0 + \beta x'|)]^{\eta} > 1$. This is akin to the behavior envisioned by *Foufoula-Georgiou et al.* [2010] (but not involving the dependence of α on slope, as in (29)), to accommodate the possibility of long-distance particle motions.

[31] We now observe the following. Consider a convex hillslope profile that is steepening with increasing x, and choose a particular starting position x'. On the one hand, we can envision a distribution of travel distances that reflects a decreasing disentrainment rate due to the steepening with increasing x, but which otherwise would reflect a constant disentrainment rate determined by the slope at the starting position x' in the absence of steepening. Note, moreover,

that this would give the appearance of a decreasing disentrainment rate with increasing travel distance r, as in (24), albeit in response to the changing surface slope rather than representing an intrinsic behavior of particle motions and their interactions with the surface independently of slope. On the other hand, we can envision a distribution of travel distances that reflects a decreasing disentrainment rate with increasing travel distance r independently of the surface steepening, as with the example above of the Pareto distribution, where the value of the parameter α is determined only by the slope at the starting position x'. Thus, these two situations may be qualitatively indistinguishable in the absence of an understanding of how surface slope affects the disentrainment rate.

[32] In each of these examples, the parametric values, μ_r and σ_r^2, of the distribution of travel distances $f_r(r; x')$ are influenced by the surface slope S. Thus, the distributions $f_r(r; x')$, obtained here from the disentrainment rate $P_r(r; x')$, describe how material, once mobilized, is dispersed downslope, where dispersal may be influenced nonlocally by changing conditions downslope. Nonetheless, this does not define a condition of nonlocal (versus local) transport as introduced above. Rather, as elaborated in the next section, the distinction between these hinges on a point of view looking upslope, which considers the source of the particles contributing to the flux of particles at a specified position x.

5. The Sediment Flux

5.1. The Flux in Convolution Form

[33] Consider the examples presented above involving the land-surface slope S. In the first case, involving a uniformly rough, planar hillslope surface with constant slope S, (11) becomes

$$q(x) = \int_{-\infty}^{x} E(x') e^{-\frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S} - 1\right)(x - x')}\, \mathrm{d}x' , \tag{31}$$

which has the form of a convolution, namely,

$$q(x) = \int_{-\infty}^{x} h_r(x - x') E(x')\, \mathrm{d}x' , \tag{32}$$

with the kernel

$$h_r(x - x') = e^{-\frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S} - 1\right)(x - x')} . \tag{33}$$

Thus, the flux at position x involves convolving the rate of mobilization $E(x')$ at all upslope positions x' with the kernel $h_r(x - x')$. With constant land-surface slope S, the kernel $h_r(x - x')$ is invariant with x' and x.

[34] In the specific case where the mobilization rate is uniform, that is, $E(x') = E$, then far enough from effects of an upper boundary at $x = x' = 0$ (which receives no material from $x' < 0$), the solution of (32) yields a uniform flux $q = E\mu_r$ with $\mu_r = \lambda_0[2S_c/(S_c - S) - 1]$ for a given slope S. That is, all positions x sufficiently far from $x = 0$ see the same total contribution of material from upslope positions, although the amount contributed from any particular upslope position x' decreases with increasing upslope distance $(x - x')$ from the position x.

[35] Imagine a set of planar hillslope surfaces whose slopes $|S|$ span the domain $[0, S_c)$, but which otherwise are subjected to identical values of the uniform mobilization rate E. The flux q at any arbitrary position x, plotted against the slope S, would yield a single-valued relation between q and S. Indeed, this plot would look like a "local" expression of transport, namely, that the flux varies as a unique function of local slope, despite the fact that the flux at any position x derives from "nonlocal" delivery of sediment to and past x from positions upslope.

[36] If, however, the mobilization rate $E(x')$ varies with position x' on a planar hillslope with constant slope S, then the convolution (32) yields different values of the flux $q(x)$ at different positions x, despite the same local slope S at these positions. This point, within the context of this section, provides a first formal view of the significance of nonlocal transport. Namely, again, imagine a set of planar hillslope surfaces whose slopes $|S|$ span the domain $[0, S_c)$, but which otherwise are subjected to identical downslope variations in the mobilization rate $E(x')$. Because in this situation, the flux has multiple values for the same surface slope, a plot of the flux q versus the slope S "sampled" from many positions x across the set of surfaces would reveal a nonunique relation between these quantities.

[37] In the second case where the surface slope $S(x')$ varies linearly with distance x', the kernel h_r looks like

$$h_r(x - x'; x') = e^{-\frac{1}{\lambda_0}\left(\frac{2S_c}{S_c - S_0 - \beta x'} - 1\right)(x - x')} . \tag{34}$$

The presence of x' in the denominator of the exponent indicates that the kernel varies with position x' in relation to the changing surface slope. This point provides a second formal view of the significance of nonlocal transport. Namely, consider two hillslope profiles, and select a position x on each with the same local slope S but with different upslope configurations of slope $S(x')$. Further, choose an upslope position x' on each profile that is the same distance from x. Because the surface slope $S(x')$ at these two positions differs, the contribution of material to the flux at x from each of these two positions x' differs. This conclusion generally applies to any two positions x' of equal upslope distance from the positions x. Hence, the convolution (32) differs between the two profiles. In general, therefore, the flux $q(x)$ is not uniquely related to the (same) surface slope $S(x)$ at the positions x. A plot of the flux q versus the slope S "sampled" from many positions x would reveal a nonunique relation between these quantities.

[38] The survival functions $R_r(r; x')$, and hence the kernels $h_r(x - x')$, in the preceding examples are based on distributions of the travel distance r that possess normal tails. Consider the possibility that the kernel derives from a heavy-tailed distribution. For example, *Foufoula-Georgiou et al.* [2010] suggest a convolution of the form

$$q(x) = -K \int_{-\infty}^{x} h_r(x - x') S(x')\, \mathrm{d}x' , \tag{35}$$

where K is a transport coefficient whose dimensions are $[\mathrm{L\ T^{-1}}]$, inasmuch as the kernel is dimensionless. In this formulation, the kernel $h_r(x - x')$ decays as a power law with $r = x - x'$. Using, for example, the form of the Pareto distribution above,

$$h_r(x - x') = \frac{r_0^{\alpha}}{(r_0 + x - x')^{\alpha}} . \tag{36}$$

Unless the shape parameter α depends on the surface slope, this kernel is independent of slope. The appearance of the slope in (35) therefore implies that the rate of mobilization of soil material is implicitly contained in the coefficient K and the slope. Namely, let $E(x') = K|S(x')|$. Then, (35) takes the form of (32). As in the preceding examples, in principle, the kernel in (35) properly weights the effect of the land-surface slope at all positions upslope of x. But this is associated with the effect of slope on the mobilization of material, not the effect of slope on the distance of downslope dispersal of this material. Moreover, the essential conclusion is the same, that the flux $q(x)$ is not uniquely related to the local slope $S(x)$ at x [*Foufoula-Georgiou et al.*, 2010]. We elaborate this conclusion in section 6.

5.2. The Flux in Advection-Diffusion Form

[39] The definition of the flux $q(x)$ as a convolution integral, (11), can be described in a simpler, perhaps more familiar, way by approximating the convolution in terms of advective and diffusive parts. Starting with (11), and noting that $x' = x - r$ so that $\mathrm{d}x' = -\mathrm{d}r$, a change of variables gives

$$q(x) = \int_0^\infty E(x-r)R_r(r; x-r)\,\mathrm{d}r\,. \qquad (37)$$

Assuming that E and R_r are continuously differentiable, we expand the integrand in (37) as a Taylor series to first order to give

$$E(x-r)R_r(r; x-r) = E(x)R_r(r; x) - \frac{\partial}{\partial x}[E(x)R_r(r; x)]r\,. \qquad (38)$$

Substituting (38) into (37) then gives

$$q(x) = E(x)\int_0^\infty R_r(r; x)\,\mathrm{d}r - \frac{\partial}{\partial x}\left[E(x)\int_0^\infty rR_r(r; x)\,\mathrm{d}r\right]. \qquad (39)$$

To proceed requires evaluating the integrals in (39).

[40] Note that by the product rule,

$$rf_r(r; x) = r\frac{\mathrm{d}F_r(r; x)}{\mathrm{d}r} = \frac{\mathrm{d}}{\mathrm{d}r}[rF_r(r; x)] - F_r(r; x)\,. \qquad (40)$$

With $R_r(r; x) = 1 - F_r(r; x)$, substitution leads to

$$rf_r(r; x) = R_r(r; x) - \frac{\mathrm{d}}{\mathrm{d}r}[rR_r(r; x)]\,. \qquad (41)$$

Integrating this from $r = 0$ to $r = \infty$,

$$\int_0^\infty rf_r(r; x)\,\mathrm{d}r = \int_0^\infty R_r(r; x)\,\mathrm{d}r - \int_0^\infty \frac{\mathrm{d}}{\mathrm{d}r}[rR_r(r; x)]\,\mathrm{d}r\,. \qquad (42)$$

According to Leibniz's rule, this may be written as

$$\int_0^\infty rf_r(r; x)\,\mathrm{d}r = \int_0^\infty R_r(r; x)\,\mathrm{d}r - \frac{\mathrm{d}}{\mathrm{d}r}\int_0^\infty rR_r(r; x)\,\mathrm{d}r\,. \qquad (43)$$

Evaluating the last integral then leads to

$$\mu_r(x) = \int_0^\infty rf_r(r; x)\,\mathrm{d}r = \int_0^\infty R_r(r; x)\,\mathrm{d}r\,, \qquad (44)$$

insofar as the limit of $rR_r(r; x)$ as $r \to \infty$ is equal to zero. This is guaranteed if $R_r(r; x)$ decays at least as fast as a negative exponential function, in which case the product $rR_r(r; x)$ looks like r/e^r, whose limit is zero as $r \to \infty$ according to l'Hôpital's rule. The absence of this condition being satisfied implies that $f_r(r; x)$ is a heavy-tailed distribution without finite mean. In turn,

$$r^2 f_r(r; x) = rR_r(r; x) - r\frac{\mathrm{d}}{\mathrm{d}r}[rR_r(r; x)]\,, \qquad (45)$$

or

$$r^2 f_r(r; x) = 2rR_r(r; x) - \frac{\mathrm{d}}{\mathrm{d}r}[r^2 R_r(r; x)]\,. \qquad (46)$$

Integrating this from $r = 0$ to $r = \infty$,

$$\int_0^\infty r^2 f_r(r; x)\,\mathrm{d}r = 2\int_0^\infty rR_r(r; x)\,\mathrm{d}r - \frac{\mathrm{d}}{\mathrm{d}r}\int_0^\infty r^2 R_r(r; x)\,\mathrm{d}r\,. \qquad (47)$$

Evaluating the last integral then leads to

$$\sigma_r^2 = \int_0^\infty r^2 f_r(r; x)\,\mathrm{d}r = 2\int_0^\infty rR_r(r; x)\,\mathrm{d}r\,, \qquad (48)$$

insofar as the limit of $r^2 R_r(r; x)$ as $r \to \infty$ is equal to zero. Again, this is guaranteed if $R_r(r; x)$ decays at least as fast as a negative exponential function, in which case the product $r^2 R_r(r; x)$ looks like r^2/e^r, whose limit is zero as $r \to \infty$ according to l'Hôpital's rule. The absence of this condition being satisfied implies that $f_r(r; x)$ is a heavy-tailed distribution without finite variance.

[41] With these definitions in place, substituting (44) and (48) into (39) then leads to an advection-diffusion form of the flux, namely,

$$q(x) = E(x)\mu_r(x) - \frac{1}{2}\frac{\partial}{\partial x}[E(x)\sigma_r^2(x)]\,, \qquad (49)$$

which we note is the result obtained by *Furbish and Haff* [2010, equation (32) therein], where r may be considered a positive or negative travel distance. At first glance, this has the appearance of a "local" expression of the flux $q(x)$ inasmuch as $E(x)$ and the moments $\mu_r(x)$ and $\sigma_r^2(x)$ are functions only of local hillslope conditions. Nonetheless, (49) is an approximation of the convolution integral (37); so in this sense, (49) is a nonlocal expression.

[42] Consider again the example involving land-surface slope, where it suffices to consider the first two cases in which the distribution of travel distances is exponential. Taking into account motions in both the positive and negative directions, the average travel distance $\mu_r(x)$ goes as

$$\mu_r(x) = -\lambda_0\left(\frac{1}{kS_p} + \frac{2}{S_c}\right)S(x) + \mathrm{O}(S^2)\,, \qquad (50)$$

where S_p is the magnitude of the surface slope beyond which all particle motion is downslope [*Furbish and Haff*, 2010] and k is a modulating factor (see section 6 below). This indicates that, at leading order, the average travel distance μ_r is proportional to the local land surface slope $S(x)$. In turn, the variance σ_r^2 of the travel distance goes as

$$\sigma_r^2 = \lambda_0^2 + \mathrm{O}(S^2)\,, \qquad (51)$$

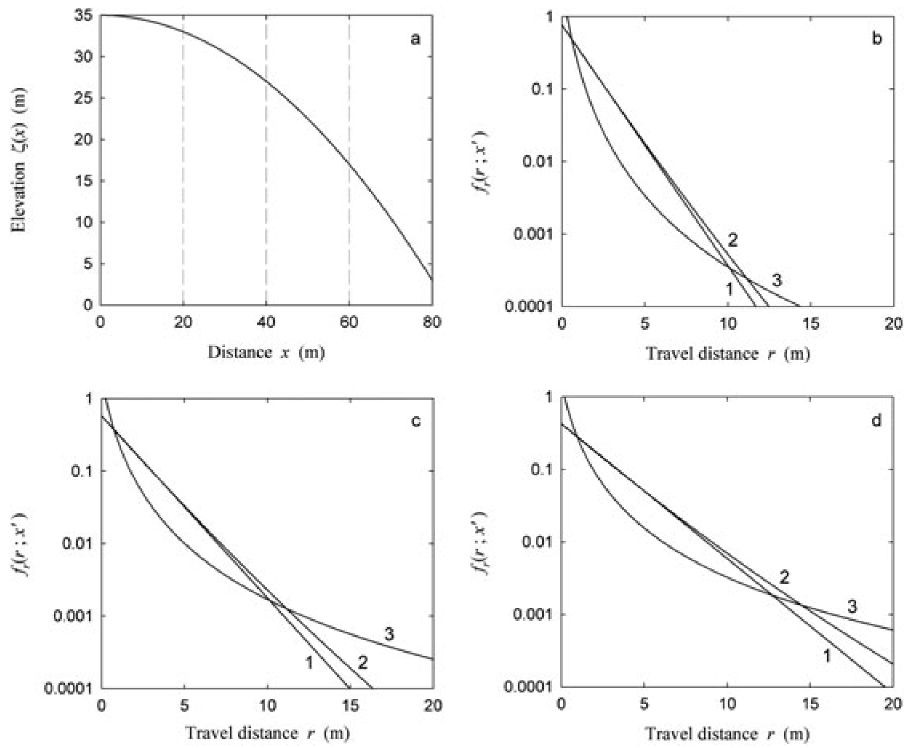

Figure 3. Example of (a) convex hillslope showing locations (dashed lines) of distributions of travel distances at (b) $x' = 20$ m, (c) $x' = 40$ m, and (d) $x' = 60$ m based on exponential distribution (21) (line 1), exponential-like distribution (24) (line 2), and Pareto distribution (28) whose shape parameter α depends on slope according to (29) (line 3). Parametric quantities are the following: $S_0 = 0$, $\beta = -0.01$ m^{-1}, $\lambda_0 = r_0 = 1$ m, $S_c = 1.5$, and $\eta = 1/2$.

which indicates that, at leading order, $\sigma_r^2(x)$ is constant. Substituting (50) and (51) into (49) gives

$$q(x) = -\lambda_0\left(\frac{1}{S_p} + \frac{2}{S_c}\right)ES + E\mathrm{O}(S^2) - \lambda_0^2\frac{\partial E}{\partial x} - \frac{\partial}{\partial x}[E\mathrm{O}(S^2)]\,. \quad (52)$$

[43] If the mobilization rate E is uniform, then in the case involving a uniformly rough, planar hillslope surface with constant slope S, the derivative terms in (52) vanish and the flux exhibits a slope-dependent relation. Moreover, consistent with the conclusions above, this looks like a "local" expression of transport, namely, that the flux varies as a unique function of local slope. If, however, the mobilization rate E varies with x, independent of slope, then the terms involving $\partial E/\partial x$ lead to a nonunique relationship between the flux and the slope.

[44] If the mobilization rate E is uniform, then for a hillslope profile involving a variation in slope $S(x)$, the last term in (52) involves the derivative of the slope S, that is, the land-surface concavity $C = \partial S/\partial x$. Similarly, if the mobilization rate $E(x)$ is a function of slope [e.g., *Foufoula-Georgiou et al.*, 2010; *Furbish and Haff*, 2010], then the last two terms in (52) involve surface concavity. In this situation, the flux exhibits a slope-concavity-dependent relation, that is, a nonunique function of local slope, as a particular value of the local slope, depending on the history of hillslope evolution, could be associated with many different values of local concavity.

[45] This raises an interesting point. Thus far, studies of transport on hillslopes have focused on how the flux is related to local slope S, whether this might involve a simple linear dependence [e.g., *Furbish et al.*, 2009a] or perhaps a nonlinear dependence [e.g., *Roering et al.*, 1999], or whether a more general formulation involving a convolution of slope is needed [*Foufoula-Georgiou et al.*, 2010; *Furbish and Haff*, 2010]. The focus on local slope represents a legacy influenced by continuum theory. But in principle, there is no reason to exclude other quantities, for example, the local concavity, in defining a local transport relation. If we insist that local transport refers to the idea that the flux can be expressed as a unique function of local slope, then the analysis above signals a nonlocal transport behavior for material dispersed over significant distances. If, however, local transport more generally refers to the idea that the flux can be related to local conditions, not restricted to slope, then (49) may be considered a "local" expression. Inasmuch as the mobilization rate E depends on slope S, then the derivatives of slope resulting from (52) represent the (nonlocal) effects of travel distances over length scales similar to, if not greater than, the length scale λ_S. Nonetheless, this requires clarification as to whether the flux is a unique function of slope and concavity; that is, the flux is a single valued function over the S-C domain. (We address this question in the next section.)

[46] Consider a heavy-tailed distribution of travel distances (and the associated kernel). Again, using the example of a Pareto distribution, inasmuch as the shape parameter $\alpha > 2$, then the moments μ_r and σ_r^2 are finite, and the flux $q(x)$ can be written as an advection-diffusion equation as in (49). However, if $1 < \alpha < 2$, then the variance σ_r^2

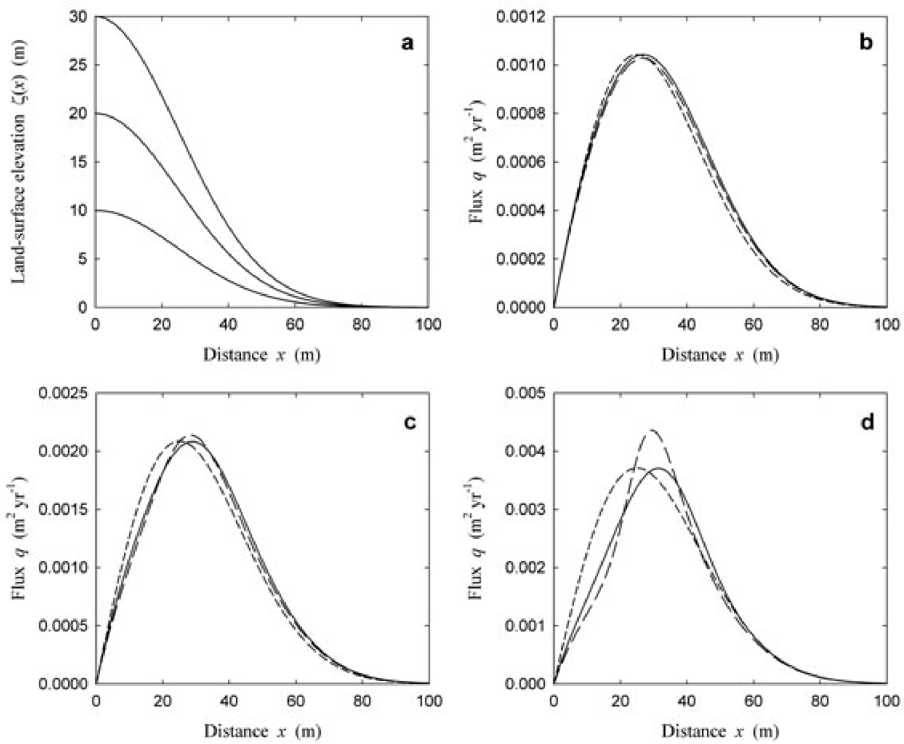

Figure 4. Plots of (a) Gaussian hillslope profiles with $\sigma_\zeta = 25$ m, and the sediment flux $q_C(x)$ (solid lines), $q_A(x)$ (long-dashed lines), and $q_L(x)$ (short-dashed lines) versus hillslope distance x for (b) $\zeta_0 = 10$ m, (c) $\zeta_0 = 20$ m, and (d) $\zeta_0 = 30$ m. Parametric quantities are the following: $\lambda_0 = 1$ m, $S_c = 1.2$, $S_p = 0.6$, $k = 1/2$, and $E = E_0 = 0.001$ m yr^{-1}.

is undefined, and the Taylor expansion used above is inappropriate. In this case, the flux $q(x)$ must be specified in its convolution form. Alternatively, returning to the formulation of *Foufoula-Georgiou et al.* [2010], these authors show that (35) can be written in the form of a fractional derivative,

$$q(x) = -K\frac{\partial^{\alpha-1}\zeta(x)}{\partial x^{\alpha-1}}\,, \tag{53}$$

where now the units of K depend on α. Note that in this formulation, α is constant and is thus independent of land-surface slope, versus possessing a slope dependence as in (29), for example. Otherwise, (35) cannot be written in the form of a fractional derivative (with specified α). As described above, this also means that the distribution of travel distances is independent of slope.

6. The Flux as a Function of Slope and Concavity

[47] As mentioned above, under certain conditions, the flux may be considered a local function of land-surface slope and concavity. To illustrate this point, we consider a Gaussian hillslope profile,

$$\zeta(x) = \zeta_0\, e^{-x^2/2\sigma_\zeta^2}\,, \tag{54}$$

where ζ_0 is the elevation of the hillslope crest at $x = 0$, and σ_ζ, like a standard deviation, sets the breadth of the profile. By varying ζ_0, we can "sample" a large number of slopes and concavities, which can be calculated analytically.

[48] We calculate the flux $q(x)$ based on a version of the convolution (11), designated as $q_C(x)$, and based on the advection-diffusion form (49), designated as $q_A(x)$. In both cases, we assume that the distribution of travel distances $f_r(r; x')$ is exponential, where the average travel distance $\mu_r(x')$ depends on the land-surface slope at x'. For comparison, we also calculate the flux based on a simple linear dependence with slope as in (2), designated as $q_L(x)$. Following *Furbish and Haff* [2010], we consider the situation in which the mobilization rate E may vary with slope as $E = E_0 + E_1|S|^m$, where E_0 and E_1 [L T^{-1}] are constants and m is an exponent. We emphasize that the calculations described next are aimed at illustrating an idea, and not at exploring effects of variations in parametric quantities.

[49] To explicitly take into account motions in both the positive and negative x directions, let $s = x' - x$ denotes a positive travel distance in the negative x direction. Then $q_C(x)$ is given by

$$q_C(x) = E\int_{-\infty}^{x} h_r(x - x')p(x')\mathrm{d}x' - E\int_{x}^{\infty} h_s(x' - x)n(x')\mathrm{d}x', \tag{55}$$

where the kernel $h_r(x - x')$ is

$$h_r(x - x') = e^{-\frac{1}{\lambda_0}\frac{S_c + S(x')}{S_c - S(x')}(x - x')}, \tag{56}$$

and the kernel $h_s(x' - x)$ is

$$h_s(x' - x) = e^{-\frac{1}{\lambda_0}\frac{S_c - S(x')}{S_c + S(x')}(x' - x)}. \tag{57}$$

Here $p(x')$ is the probability that particles, once mobilized, move in the positive x direction and is specified as $p(x') = (1/2)\exp(-S/kS_p)$ for $S > 0$ or $p(x') = 1 - (1/2)\exp(-|S|/kS_p)$ for $S \leq 0$, and $n(x') = 1 - p(x')$ is the probability that particles move in the negative x direction, where k is a factor that sets the rate of transition from $p(x') = n(x') = 1/2$ at $S = 0$ to $p(x') = 0$ or $p(x') = 1$ as $|S| \to S_p$. This is a modification of the linear variation in $p(x')$ and $n(x')$ between $S = 0$ and $|S| = S_p$ presented in *Furbish and Haff* [2010] and has the form of the model suggested by *Dunne et al.* [2010] for rain splash transport. To integrate (55) numerically, we choose a Riemann interval $\Delta x \ll \lambda_0$.

[50] To calculate $q_A(x)$, we use (49) where $\mu_r(x) = p(x)\lambda_r - n(x)\lambda_s$ and $\sigma_r^2(x) = p(x)[\lambda_r(x)]^2 + n(x)[\lambda_s(x)]^2$. Here λ_r and λ_s are the average travel distances in the positive and negative x directions and are equal to

$$\lambda_r = \lambda_0 \frac{S_c - S(x)}{S_c + S(x)} \tag{58}$$

and

$$\lambda_s = \lambda_0 \frac{S_c + S(x)}{S_c - S(x)}. \tag{59}$$

[51] To calculate $q_L(x)$, we use (2) and choose a value of the diffusion-like coefficient D so that the maximum value of $q_L(x)$ is approximately the same as the calculated maximum of $q_C(x)$. This is only for visual comparison of the flux curves.

[52] In calculating $q_C(x)$ and $q_A(x)$, we extend the calculations over both the positive and negative x domains but show only the positive x domain in Figures 4 and 5. This amounts to a reflection across $x = 0$ to satisfy the zero-flux condition at this position. In addition, we extend the calculations to large $\pm x$, well beyond the domain shown in the figures, to ensure that any effects of the computational boundaries at large $\pm x$ are negligible.

[53] For a uniform mobilization rate $E = E_0$ (and $E_1 = 0$), then with small relief ζ_0 such that average travel distances are not too large, the advection-diffusion approximation (49) closely matches the full convolution (55) (Figure 4). That is, the Taylor expansion leading to (49) with small r is reasonable. Moreover, the calculated fluxes $q_C(x)$ and $q_A(x)$ are similar to the simple linear model $q_L(x)$. With increasing relief (and therefore increasing average travel distances), the maximum flux shifts downslope relative to the linear model. As well, the approximation (49) increasingly deviates from the convolution (55).

[54] For a nonuniform mobilization rate $E = E_0 + E_1|S|^m$, then with small relief ζ_0, the advection-diffusion approximation (49) again closely matches the full convolution (55) (Figure 5). The flux near the hillslope crest increases less rapidly with x than does the linear model due to the negative land-surface concavity and the maximum flux shifts downslope. With increasing relief, the approximation (49) increasingly deviates from the convolution (55).

[55] As anticipated, a plot of q_C (or q_A) versus the land-surface slope $|S|$ reveals a nonunique relationship between these quantities (Figure 6). In this plot, each loop represents a set of positions x "sampled" from a profile with specified relief ζ_0 and associated ranges of slope and concavity. Increasing loop size coincides with increasing ζ_0. Choosing any particular value of slope $|S|$, the smaller of the two calculated fluxes on a loop coincides with the higher of the two positions x on the profile possessing this slope. The larger of the two fluxes coincides with the lower position x possessing this slope. The larger flux reflects a longer and generally steeper (and concave), upslope distance contributing to the flux.

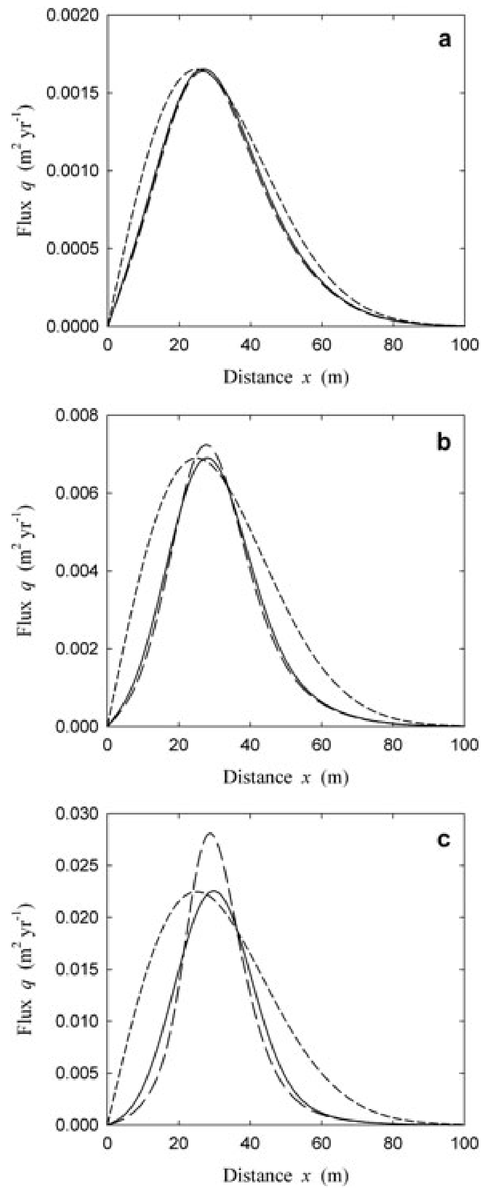

Figure 5. Plots of the sediment flux $q_C(x)$ (solid lines), $q_A(x)$ (long-dashed lines), and $q_L(x)$ (short-dashed lines) versus hillslope distance x for (a) $\zeta_0 = 10$ m, (b) $\zeta_0 = 20$ m, and (c) $\zeta_0 = 30$ m. Parametric quantities are the following: $\lambda_0 = 1$ m, $S_c = 1.2$, $S_p = 0.6$, $k = 1/2$, and $E = E_0 + E_1|S|^m$ with $E_0 = 0.001$ m yr^{-1}, $E_1 = 0.01$ m yr^{-1}, and $m = 2$.

[56] Note that the specific shapes of the loops in Figure 6 may vary with the form of the hillslope profile. Indeed, the points composing these loops would collectively appear as single-valued curves in the case of profiles that are entirely convex from crest to foot, or entirely concave from

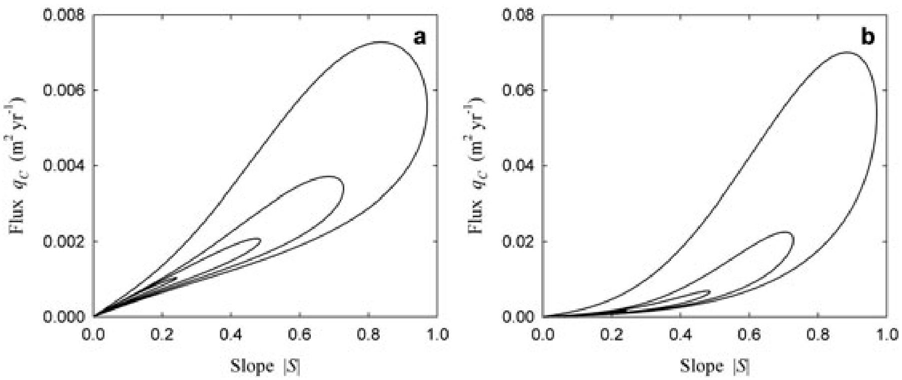

Figure 6. Plots of sediment flux q_C versus slope $|S|$ using (a) parametric quantities in Figure 4 with $E = E_0$ and (b) parametric quantities in Figure 5 with $E = E_0 + E_1|S|^m$. Loop sizes, from smallest to largest, coincide with ζ_0 = 10 m, ζ_0 = 20 m, ζ_0 = 30 m, and ζ_0 = 40 m.

crest to foot [e.g., *Furbish and Haff*, 2010, Figure 11]. We further note that these loops are smooth curves as a consequence of the smooth Gaussian profiles used to calculate the fluxes. If, instead, the calculations were based on a set of realistic convex-concave profiles of varying relief within, say, a large catchment area, then the points composing the loops in Figure 6 likely would appear as a cloud of points roughly enclosed by the largest loop, entirely akin to the cloud of points calculated using (3) in Figure 9 of *Roering et al*. [1999].

[57] Consider the expression (52). With uniform $E = E_0$ ($\partial E/\partial x = 0$), then inasmuch as the advective term at lowest order goes with $|S|$, and the diffusive term goes with the derivative of S^2; that is, with $2S\partial S/\partial x \sim SC$, this suggests that the flux ought to exhibit an approximately single-valued relationship over the $|S| - C$ domain (Figure 7a) or over the $|S| - |S|C$ domain (Figure 7b) for slopes (and travel distances) that are not too large. Rotation of these plots (not shown) visually reveals that this is the case. One may therefore consider (49) as representing an approximate, local formulation of the flux, albeit involving the land-surface concavity as well as the slope. We note, however, that this interpretation is more complicated if the rate of mobilization E varies with slope.

7. A Matter of Scale

[58] Returning to the physical distinction between local versus nonlocal transport, we note that this distinction hinges on the scale of particle motions relative to the scale of resolution of the quantities—surface slope, for example—to which the motions are functionally related. Envision, for example, an observer having the stature (< 1 cm) of the children in the 1989 Disney film, "Honey, I Shrunk the Kids," standing at position x on a smooth, sloping granular surface in a rainstorm. To such an observer, the sediment flux past x due to rain splash is determined by particle motions produced by raindrop impacts at all positions $x' = x - r$ upslope and downslope of x within reach of the splash distances r. Moreover, this observer could justifiably formulate the downslope part of this flux in terms of a convolution integral having the form of (11) (which can be generalized to accommodate both downslope and upslope motions). At this scale, the formulation looks like a nonlocal one, inasmuch as the flux at x depends on the rate of particle mobilization and the distribution of travel distances determined at positions "far" from x. Nonetheless, in this problem, the distribution of splash distances to an "unshrunken" observer is conventionally attributed to the surface slope measured at a scale larger than the mean splash distance [e.g., *Furbish et al.*, 2009a;

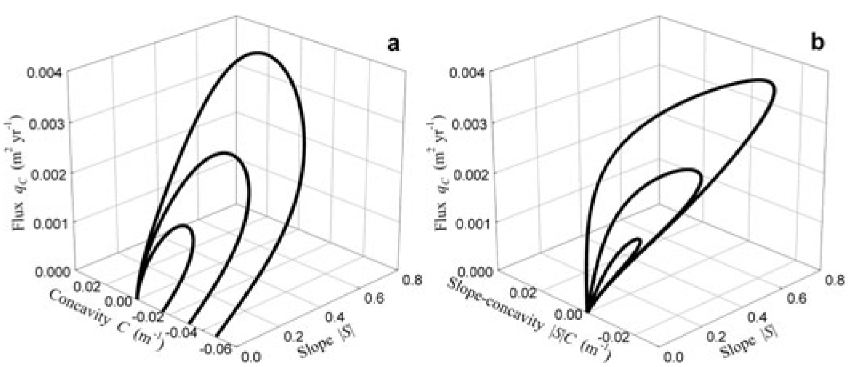

Figure 7. Plot of sediment flux q_C versus (a) slope $|S|$ and concavity C and (b) slope $|S|$ and slope-concavity product $|S|C$. Parametric quantities are the same as in Figure 4. Loop sizes, from smallest to largest, coincide with ζ_0 = 10 m, ζ_0 = 20 m, and ζ_0 = 30 m.

Dunne et al., 2010]. In zooming out to this larger scale, variations in the mean travel distance are associated with variations in surface slope, consistent with a local formulation of the flux described by the advective part of (49). And, any detailed effects on splash distances due to microtopographic variations at a scale smaller than the travel distances represent noise.

[59] More generally, if a factor that "controls" transport (e.g., surface slope or mobilization rate) is defined or measured at a specified resolution Δx, then motions smaller than this resolution cannot be attributed to the controlling factor as manifest at a scale smaller than Δx. That is, details of motions and the effect of the factor on these motions at a scale smaller than Δx cannot be resolved or used as the basis of a formulation of the flux. The smallest scale of the controlling factor is that defined by Δx, and the only possibility is a local formulation of the flux, as with the example of rain splash above. In turn, for motions larger than Δx, then a nonlocal formulation is imaginable. That is, because motions are larger than the resolution of the controlling factor, the serial (upslope and downslope) configuration of the factor may be important in determining the flux at any position x.

8. Discussion and Conclusions

[60] The topic of local versus nonlocal formulations of the sediment flux is one for which the theory [*Foufoula-Georgiou et al.*, 2010; *Furbish and Haff*, 2010; *Tucker and Bradley*, 2010; this paper] has far outpaced experimental and field-based observations [e.g., *Dunne et al.*, 2010; *Gabet and Mendoza*, 2012; *Ghahramani et al.*, 2012; *Cooper et al.*, 2012] needed to inform the theory. This is not necessarily a bad thing. Theoretical formulations often precede an empirical basis. But at risk is bringing to bear a sophisticated mathematics that is not supported by the stochastic nature, and uncertainty, of the processes involved [*Callander*, 1978]. As summarized above and described more fully in *Furbish and Haff* [2010], certain quantities in the formulation (e.g., the characteristic length scale λ_0 and the critical slopes S_c and S_p) have a physical basis supported by empirical evidence. But experimental and field-based measurements of mobilization rates and details of travel distances are difficult to obtain, notably given the intermittency of many hillslope transport processes and the slow rates of change in hillslope morphology. As with previous formulations of the sediment flux on hillslopes—a legacy beginning with the work of W. E. H. Culling—this is a problem for which, in order to "test" a specific nonlocal formulation, we must appeal to consistency between measured hillslope configurations and predicted hillslope configurations predicated on the proposed nonlocal formulation and on sufficient knowledge of initial and boundary conditions. Given that we really never know with certainty the initial and boundary conditions belonging to the hillslopes whose surfaces we are surveying, we further suggest that, in order to test alternative transport models, there may be value in using ancillary quantities, such as tracers, whose behavior is coupled with the geometric evolution of hillslopes, thereby adding discriminatory information beyond that provided by land-surface elevation alone [*Furbish*, 2003; *Roering et al.*, 2004].

[61] These points highlight the need to ponder the sediment disentrainment process. We have focused here on the influence of land-surface slope in order to illustrate the functional attributes of local versus nonlocal transport, assuming that slope exerts a primary control on the likelihood of disentrainment. But in detail, the disentrainment process also must involve effects of sediment particle size, surface roughness, and particle-surface momentum exchanges that influence how the distribution of travel distances varies with increasing surface steepness [e.g., *Gabet and Mendoza*, 2012]. On the other hand, it remains an open question whether this level of detail can be discerned in the behavior of hillslope evolution models at timescales significantly longer than the soil residence time, or from field measurements of hillslope conditions, which effectively are snapshots of hillslope behavior. What seems clear is that, for given external conditions, hillslope geometries predicted using nonlocal formulations of transport differ from geometries based on local formulations. Specifically, whereas local slope-dependent formulae, such as (2), yield a parabolic hillslope profile under steady state conditions [e.g., *Fernandes and Dietrich*, 1997; *Anderson*, 2002], current nonlocal formulations suggest that steady state profiles have nonparabolic forms that generally become increasingly linear with increasing distance from the hillslope crest [*Foufoula-Georgiou et al.*, 2010; *Furbish and Haff*, 2010; *Tucker and Bradley*, 2010]. What needs clarification is the sensitivity of hillslope geometries to the factors that control the sediment mobilization rate and to the specific form of the distribution of particle travel distances, used in nonlocal formulations—including how these hillslope geometries might vary under transient conditions.

[62] Whereas local formulations of transport are reasonable when particle motions involve small length scales associated with creation and collapse of porosity within the soil column or with rain splash, nonlocal formulations may be more appropriate in steeplands where patchy, intermittent motions following disturbances involve large travel distances. With this in mind, our principal conclusions are the following: (1) Time-continuous formulations of the sediment flux must involve time averaging to accommodate the patchy, intermittent nature of transport processes in many steepland settings. (2) Nonlocal formulations of the sediment flux that involve a convolution of hillslope surface (slope) conditions contain a kernel that derives from a probabilistic description of sediment disentrainment following mobilization. This description provides a unifying basis for explaining how different distributions of particle travel distances can arise in relation to the disentrainment process. The form of the kernel therefore characterizes whether travel distances depend on particle behavior unconditioned by changing (downslope) surface conditions [e.g., *Foufoula-Georgiou et al.*, 2010], or on surface conditions at the position where motions originate [e.g., *Furbish and Haff*, 2010], or on the changing surface conditions experienced by particles during their downslope motions [e.g., *Tucker and Bradley*, 2010]. (3) A nonlocal formulation of the flux can be approximated as an advection-diffusion equation if the first two moments (the mean and variance) of the distribution of travel distances are finite—although the adequacy of this approximation decreases with increasing travel distances. In general, nonlocal formulations lead to a nonunique relationship between the flux and the local surface slope. (4) A nonlocal formulation of the flux is

relevant when particle motions are larger than the scale of resolution at which the factors controlling the transport are defined or measured. In this situation, the serial (upslope or downslope) configuration of the factors (e.g., surface slope) may be important in determining the flux at any position x.

Notation

- A vertical surface through which transport occurs [L^2].
- b contour width over which transport occurs [L].
- B function defined as $B(x') = S_C - S_0 - bx'$.
- C land-surface concavity defined as $C = \partial^2\zeta/\partial x^2$ [L^{-1}].
- D diffusion-like coefficient [L^2 T^{-1}].
- E rate of mobilization of soil material [L T^{-1}].
- f_r probability density function of travel distances r [L^{-1}].
- F_r cumulative distribution function of travel distances r.
- h local active soil thickness [L].
- h_r, h_s kernels in convolution of the mobilization rate.
- $\mathbf{H}$ vector of local hillslope quantities.
- k factor modulating the slope S_p.
- K transport coefficient [L T^{-1}].
- m exponent associated with slope-dependent mobilization rate.
- n probability that particles move in negative x direction.
- $\mathbf{n}$ unit vector normal to A.
- N total number of particles.
- p probability that particles move in positive x direction.
- P_r spatial disentrainment rate [L^{-1}].
- q sediment flux [L^2 T^{-1}].
- Q sediment volume discharge [L^3 T^{-1}].
- q_C flux calculated from convolution [L^2 T^{-1}].
- q_A flux calculated from advection-diffusion approximation of convolution [L^2 T^{-1}].
- q_L flux calculated from linear dependence on slope [L^2 T^{-1}].
- r travel distance parallel to x [L].
- r_0 location parameter in Pareto distribution [L].
- R_r survival function defined as $1 - F_r$.
- s positive travel distance in the negative x direction [L].
- S local land-surface slope.
- S_c magnitude of critical slope beyond which particle motions continue indefinitely.
- S_p magnitude of critical slope beyond which particle motions are entirely downslope.
- S_0 land-surface slope at hillslope crest.
- t time [T].
- t_0 initial time [T].
- T averaging interval [T].
- u_p surface-normal particle velocity [L T^{-1}].
- $\mathbf{u}_p$ particle velocity field at surface A [L T^{-1}].
- w variable of integration.
- x, y, z Cartesian coordinate axes [L].
- α shape parameter in Pareto distribution.
- β rate of change in land-surface slope [L^{-1}].
- γ exponent in relation defining heavy-tailed distribution.
- λ_r, λ_s average travel distances in positive and negative x directions [L].
- λ_S characteristic length scale [L].
- λ_0 characteristic travel distance on horizontal surface [L].
- μ_r average travel distance [L].
- σ_r^2 variance of travel distances [L^2].
- σ_ζ quantity determining breadth of Gaussian hillslope profile [L].
- ζ land-surface elevation [L].
- ζ_0 elevation of hillslope crest at $x = 0$ [L].
- η exponent in relation of α to land-surface slope.

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