NEUROBIOLOGY EQUATIONS

1. Membrane potential ($V_m$)

$$ V_m = V_{in} - V_{out} $$

where $V_m$ is the membrane potential, $V_{in}$ is the potential on the inside of the cell, and $V_{out}$ is the potential on the outside of the cell.

2. Ohm’s Law (and derivations)

$$ V = IR $$

where $V$ is voltage (measured in volts), $I$ is current (measured in amperes or amps), and $R$ is resistance (measured in ohms).

Because conductance is the inverse of resistance ($R$), Ohm’s law can be rewritten as:

$$ V = I/g $$

where $g$ is conductance (measured in mhos or Siemens).

The most frequently used derivation of Ohm’s Law in neurobiology rearranges the terms to measure current with respect to conductance and voltage:

$$ I_x = g_x (V_m - E_x) $$

Where $I_x$ is the current flow for ion $X$, $g_x$ is the conductance for ion $X$, $V_m$ is membrane potential, $E_x$ is the Nernst or Equilibrium Potential for ion $X$, and $V_m - E_x$ is the driving force for ion $X$.

3. Nernst Equation (also known as the Nernst Potential or the Equilibrium Potential)

$$ E_x = \frac{RT}{zF} \ln \frac{[X]_{out}}{[X]_{in}} $$

where $E_x$ is the equilibrium potential for ion $X$, $R$ is the gas constant, $T$ is temperature (degrees Kelvin), $z$ is the valence of ion $X$, and $[X]_{out}$ and $[X]_{in}$ are the concentrations of ion $X$ outside and inside the cell, respectively.
Since \( RT/F \) is 25 mV at 25°C (room temperature), and the constant for converting from natural logarithm to base 10 logarithm is 2.3, the Nernst equation can be written as:

\[
E_x = \frac{58 \text{ mV}}{z} \log \frac{[X]_{\text{out}}}{[X]_{\text{in}}}
\]

4. Goldman Equation (for the resting membrane potential)

\[
V_m = \frac{58 \text{ mV}}{z} \log \frac{P_K [K]_{\text{out}} + P_{Na} [Na]_{\text{out}} + P_{Cl} [Cl]_{\text{in}}}{P_K [K]_{\text{in}} + P_{Na} [Na]_{\text{in}} + P_{Cl} [Cl]_{\text{out}}}
\]

where \( P_K, P_{Na} \) and \( P_{Cl} \) = permeabilities for \( K^+ \), \( Na^+ \) and \( Cl^- \) ions, respectively.

5. Capacitance

\[ Q = CV \]

where \( Q \) is charge in coulombs, \( C \) is capacitance in farads, and \( V \) is voltage in volts.

Because current (I) is the flow of charge per unit time, then current flow across a capacitor can be determined by the derivative of the above equation:

\[ I = \frac{dQ}{dt} = C \frac{dV}{dt} \]

6. Time constant (tau \( \tau \); the time required for the membrane potential to rise to 63% of its steady state value)

The time constant is obtained from the equation that describes the rising phase of the change in membrane potential over time:

\[
\frac{dV_m}{dt} = I_m R_m \left( 1 - e^{-t/\tau} \right)
\]

where \( \frac{dV_m}{dt} \) is the rate of change in membrane potential \( (V_m) \) over time, \( I_m \) is membrane current, \( R_m \) is membrane resistance, \( t \) is time, and \( \tau \) is the time constant. \( \tau \) is further defined as:

\[ \tau = R_m C_m \]

where \( R_m \) is membrane resistance and \( C_m \) is membrane capacitance.
7. **Length constant (lambda, \( \lambda \); the distance along the membrane for membrane potential to decay to 37\% (1/e) of its original value)**

The length constant is obtained from the equation that describes the decay in the amplitude of the membrane potential over distance to a current injection at point 0:

\[
V_{m_x} = V_{m_0} e^{-x/\lambda}
\]

where \( V_{m_x} \) is the membrane potential at distance \( x \), \( V_{m_0} \) is the membrane potential at the site of current injection (\( x = 0 \)), \( x \) is distance from the site of current injection, and \( \lambda \) is the length constant. \( \lambda \) is further defined as:

\[
\lambda = \sqrt{R_m/R_i}
\]

Where \( R_m \) is membrane resistance and \( R_i \) is internal or cytoplasmic resistance.